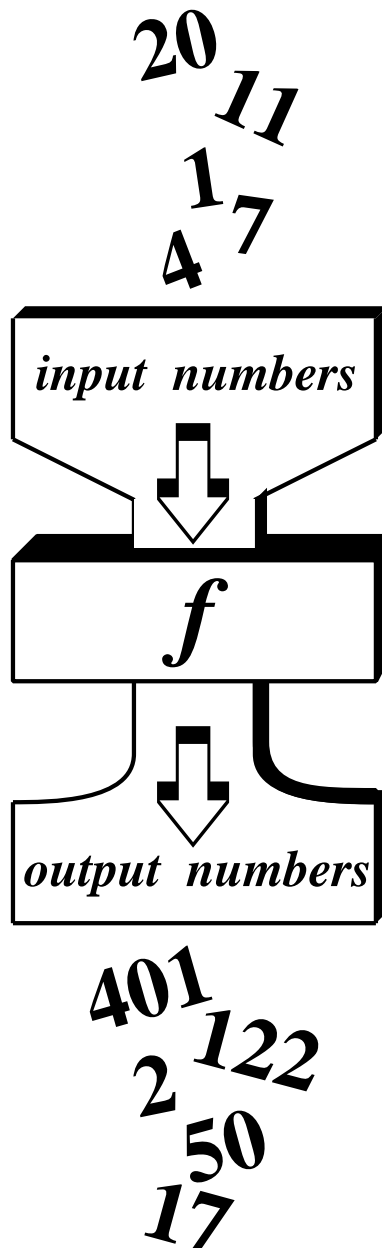


# FUNCTIONS



# Functions

## Chapter 1.

## Functions : GCSE

### 1.1 Introduction : What is a function ?

At a simple level, a function can be thought of as a mathematical rule that 'does things to numbers'.

If it is helpful, a function can be given a name.

Here is a description of the function that I've called  $f$ .

$$f(x) = x^2 + 1$$

Now, suppose I have the number 4.

What does function  $f$  do to the number 4 ?



### 1.2 An example of an examination function question.

Consider the function,  $f(x) = x^2 + 1$

Calculate,

(i)  $f(7)$

(ii)  $f(1)$

(iii)  $f(11)$

(iv)  $f(20)$

(v)  $f(5)$

(vi)  $f(-5)$

(vii)  $f(6)$

(viii)  $f(-6)$

#### NOTE:

$$f(\text{input number}) = \text{output number}$$

The numbers that are allowed into  $f$  is the function's **domain**.

The numbers that can come out of  $f$  is the function's **range**.

### 1.3.A Exercise

#### Question 1.

If  $f(x) = 4x + 13$ , find;

(i)  $f(1)$                       (ii)  $f(2)$                       (iii)  $f(3)$

(iv)  $f(0)$                       (v)  $f(10)$                       (vi)  $f(-2)$

#### Question 2.

If  $g(x) = \frac{1}{2}x^2$ , find;

(i)  $g(1)$                       (ii)  $g(-1)$                       (iii)  $g(4)$

(iv)  $g(10)$                       (v)  $g(0)$                       (vi)  $g(-3)$

#### Question 3.

If  $h(x) = 7 + \sqrt{x}$ , with domain  $x \geq 0$ , find;

(i)  $h(100)$                       (ii)  $h(4)$                       (iii)  $h(81)$

(iv)  $h(0)$                       (v)  $h(400)$                       (vi)  $h(121)$

Explain why the restriction is placed upon the domain that only numbers greater than or equal to zero can be allowed into this function.

**Question 4.**

If  $m(x) = 14 + 3x$ , find;

(i)  $m(1)$                       (ii)  $m(-1)$                       (iii)  $m(6)$

(iv)  $m(9)$                       (v)  $m(-3)$                       (vi)  $m(-5)$

**Question 5.**

If  $n(x) = 10 - 2x$ , find;

(i)  $n(4)$                       (ii)  $n(0)$                       (iii)  $n(-2)$

(iv)  $n(11)$                       (v)  $n(-8)$                       (vi)  $n(\frac{1}{2})$

**Question 6.**

Consider the function,

$$d(x) = \frac{6}{x} \text{ with domain } x \neq 0.$$

Find;

(i)  $d(3)$                       (ii)  $d(6)$                       (iii)  $d(12)$

(iv)  $d(-2)$                       (v)  $d(0.1)$                       (vi)  $d(-0.1)$

Explain why the number zero is not allowed into this function.

**Question 7.**

If  $e(x) = \sqrt{4x + 1}$ , with domain  $x \geq -0.25$  find;

(i)  $e(0)$                       (ii)  $e(2)$                       (iii)  $e(6)$

(iv)  $e(12)$                       (v)  $e\left(\frac{3}{4}\right)$                       (vi)  $e(-0.25)$

**Question 8.**

If  $k(x) = 4x^2 + 3$ , find;

(i)  $k(1)$                       (ii)  $k(-1)$                       (iii)  $k(2)$

(iv)  $k(-2)$                       (v)  $k(5)$                       (iv)  $k(-5)$

**Question 9.**

If  $A(x) = 2x^2 + 3x - 1$ , find;

(i)  $A(1)$                       (ii)  $A(0)$                       (iii)  $A(5)$

(iv)  $A(10)$                       (v)  $A(100)$                       (iv)  $A(-1)$

**Question 10.**

If  $H(x) = (2x + 5)(3x + 4)$ , find;

(i)  $H(1)$                       (ii)  $H(0)$                       (iii)  $H(-2)$

(iv)  $H(-1)$                       (v)  $H(2)$                       (vi)  $H(-3)$

**Question 11.**

This question is about a mystery function,  $M$ .

We can drop numbers into  $M$ , and see what it does to those numbers.

If I drop the numbers 0, 1, 2, 3, 4 and 5 into  $M$  then 1, 4, 7, 10, 13 and 16 come out.

In other words     $M(0) = 1$   
                           $M(1) = 4$   
                           $M(2) = 7$   
                           $M(3) = 10$   
                           $M(4) = 13$   
                          and     $M(5) = 16$

Suggest a rule that  $M$  is applying.

*It must turn all five input numbers into the five output numbers.*

*If your rule only works on one or two numbers, it's no good !*

**Question 12.**

Here is another 'guess the rule' question.

$N(0) = -1$   
 $N(1) = 0$   
 $N(2) = 3$   
 $N(3) = 8$   
 $N(4) = 15$   
and     $N(5) = 24$

Suggest a rule that  $N$  is applying.

### 1.3.B Exercise (Harder Questions)

#### Question 1.

If  $f(x) = 13x + 5$ , find;

(i)  $f(1)$                       (ii)  $f(-1)$                       (iii)  $f(2)$

(iv)  $f(10)$                       (v)  $f(0)$                       (vi)  $f(-3)$

#### Question 2.

If  $g(x) = 17 - 2x$ , find;

(i)  $g(7)$                       (ii)  $g(10)$                       (iii)  $g(17)$

(iv)  $g(50)$                       (v)  $g(-1)$                       (vi)  $g(-8)$

#### Question 3.

If  $h(x) = \sqrt{101 - x}$ ,  $x \leq 101$ . find;

(i)  $h(1)$                       (ii)  $h(100)$                       (iii)  $h(65)$

(iv)  $h(52)$                       (v)  $h(20)$                       (vi)  $h(-20)$

(vii) Try to find  $h(105)$

Hence explain the restriction on the input (domain) that  $x \leq 101$ .

**Question 4.**

If  $k(x) = 10x^2 + 11$ , find;

(i)  $k(2)$                       (ii)  $k(-2)$                       (iii)  $k(3)$

(iv)  $k(-3)$                       (v)  $k(5)$                       (iv)  $k(-5)$

**Question 5.**

If  $L(x) = 4x^2 + 5x + 7$ , find;

(i)  $L(1)$                       (ii)  $L(3)$                       (iii)  $L(10)$

(iv)  $L(-1)$                       (v)  $L(7)$                       (iv)  $L(-3)$

**Question 6.**

If  $m(x) = x^3 + x^2 + x$ , find;

(i)  $m(2)$                       (ii)  $m(3)$                       (iii)  $m(-1)$

(iv)  $m(10)$                       (v)  $m(5)$                       (iv)  $m(-3)$



**Question 7.**

If  $n(x) = \frac{24}{x}$ ,  $x \neq 0$ , find;

(i)  $n(6)$                       (ii)  $n(8)$                       (iii)  $n(24)$

(iv)  $n(-4)$                       (v)  $n(48)$                       (vi)  $n(0.5)$

(vii) Try to find  $n(0)$   
Hence explain the restriction on the domain that  $x \neq 0$ .

**Question 8.**

If  $p(x) = \sqrt{3x}$ ,  $x \geq 0$  find;

(i)  $p(3)$                       (ii)  $p(12)$                       (iii)  $p(27)$

(iv)  $p(48)$                       (v)  $p(\frac{1}{3})$                       (vi)  $p(\frac{25}{3})$

(vii) Try to find  $p(-3)$   
Hence explain the restriction on the input that  $x \geq 0$ .

**Question 9.**

If  $q(x) = x^3 + x$ , find;

(i)  $q(4)$                       (ii)  $q(\frac{1}{2})$                       (iii)  $q(\frac{1}{10})$

**Question 10.**

If

$$r(x) = x^{\frac{1}{2}}, \quad x \geq 0.$$

find;

(i)  $r(25)$                       (ii)  $r(81)$                       (iii)  $r\left(\frac{1}{4}\right)$

(iv) Explain the restriction on the domain that  $x \geq 0$ .

**Question 11.**

If

$$s(x) = x^{-2}, \quad x \neq 0.$$

find;

(i)  $s(2)$                       (ii)  $s(5)$                       (iii)  $s(4)$

(iv) Explain the restriction on the input that  $x \neq 0$ .

**Question 12.**

If

$$t(x) = x^{\frac{1}{2}} + x, \quad x \geq 0.$$

find;

(i)  $t(9)$                       (ii)  $t(36)$                       (iii)  $t(121)$

(iv) Explain the restriction on the domain that  $x \geq 0$ .

**Question 13.**

If

$$u(x) = x^{-1} + 1, \quad x \neq 0.$$

find;

**(i)**  $u(3)$

**(ii)**  $u(10)$

**(iii)**  $u(-10)$

**(iv)**  $u(5)$

**(v)**  $u(-5)$

**(iv)**  $u(-100)$

**Question 14.**If  $f(x) = x^2$  find;

**(i)**  $f(z+1)$

**HINT:** Use FOIL to work out  $(z+1)(z+1)$  in algebra.

**(ii)**  $f(z+5)$

**HINT:** Use FOIL to work out  $(z+5)(z+5)$  in algebra.

**(iii)**  $f(2z+3)$

**Question 15.**

If  $f(x) = x + 1$  find;

(i)  $f(3)$

(ii)  $f(-3)$

(iii)  $f(w^2)$

(iv)  $f(w^2 + w)$

**Question 16.**

If  $f(x) = x^2 + x$  find;

(i)  $f(4)$

(ii)  $f(-4)$

(iii)  $f(z + 1)$

(iv)  $f(z^2 + z)$

## 1.4 Answers

### 1.4.1 Solutions (Introductory Example)



$$f(4) = 17$$

- |       |               |        |               |
|-------|---------------|--------|---------------|
| (i)   | $f(7) = 50$   | (ii)   | $f(1) = 2$    |
| (iii) | $f(11) = 122$ | (iv)   | $f(20) = 401$ |
| (v)   | $f(5) = 26$   | (vi)   | $f(-5) = 26$  |
| (vii) | $f(6) = 37$   | (viii) | $f(-6) = 37$  |

### 1.4.2 Solutions (1.3.A Exercise)

#### Answer 1.

- |      |    |      |    |       |    |
|------|----|------|----|-------|----|
| (i)  | 17 | (ii) | 21 | (iii) | 25 |
| (iv) | 13 | (v)  | 53 | (vi)  | 5  |

#### Answer 2.

- |      |     |      |     |       |     |
|------|-----|------|-----|-------|-----|
| (i)  | 0.5 | (ii) | 0.5 | (iii) | 8   |
| (iv) | 50  | (v)  | 0   | (vi)  | 4.5 |

#### Answer 3.

- |      |    |      |    |       |    |
|------|----|------|----|-------|----|
| (i)  | 17 | (ii) | 9  | (iii) | 16 |
| (iv) | 7  | (v)  | 27 | (vi)  | 18 |

Domain restricted to avoid trying to take the square root of a negative number.

#### Answer 4.

- |      |    |      |    |       |    |
|------|----|------|----|-------|----|
| (i)  | 17 | (ii) | 11 | (iii) | 32 |
| (iv) | 41 | (v)  | 5  | (vi)  | -1 |

#### Answer 5.

- |      |     |      |    |       |    |
|------|-----|------|----|-------|----|
| (i)  | 2   | (ii) | 10 | (iii) | 14 |
| (iv) | -12 | (v)  | 26 | (vi)  | 9  |

#### Answer 6.

- |      |    |      |    |       |     |
|------|----|------|----|-------|-----|
| (i)  | 2  | (ii) | 1  | (iii) | 0.5 |
| (iv) | -3 | (v)  | 60 | (vi)  | -60 |

Domain is restricted to avoid division by zero, which has no meaning in mathematics.

#### Answer 7.

- |      |   |      |   |       |   |
|------|---|------|---|-------|---|
| (i)  | 1 | (ii) | 3 | (iii) | 5 |
| (iv) | 7 | (v)  | 2 | (vi)  | 0 |

**Answer 8.**

- |        |    |        |     |         |     |
|--------|----|--------|-----|---------|-----|
| ( i )  | 7  | ( ii ) | 7   | ( iii ) | 19  |
| ( iv ) | 19 | ( v )  | 103 | ( vi )  | 103 |

**Answer 9.**

- |        |     |        |        |         |     |
|--------|-----|--------|--------|---------|-----|
| ( i )  | 4   | ( ii ) | - 1    | ( iii ) | 64  |
| ( iv ) | 229 | ( v )  | 20 299 | ( vi )  | - 2 |

**Answer 10.**

- |        |    |        |    |         |     |
|--------|----|--------|----|---------|-----|
| ( i )  | 49 | ( ii ) | 20 | ( iii ) | - 2 |
| ( iv ) | 3  | ( v )  | 90 | ( vi )  | 5   |

**Answer 11.**

$$M(x) = 3x + 1$$

**Answer 12.**

$$N(x) = x^2 - 1$$

### 1.4.3 Solutions (1.3.B Exercise (Harder Questions))

#### Answer 1.

- |      |     |      |    |       |     |
|------|-----|------|----|-------|-----|
| (i)  | 18  | (ii) | -8 | (iii) | 31  |
| (iv) | 135 | (v)  | 5  | (vi)  | -34 |

#### Answer 2.

- |      |     |      |    |       |     |
|------|-----|------|----|-------|-----|
| (i)  | 3   | (ii) | -3 | (iii) | -17 |
| (iv) | -83 | (v)  | 19 | (vi)  | 33  |

#### Answer 3.

- |      |    |      |   |       |    |
|------|----|------|---|-------|----|
| (i)  | 10 | (ii) | 1 | (iii) | 6  |
| (iv) | 7  | (v)  | 9 | (vi)  | 11 |

Domain is all real numbers such that  $x \leq 101$  because otherwise you could try to take the square root of a negative number which is not real.

It could crash your computer program, for example.

#### Answer 4.

- |      |     |      |     |       |     |
|------|-----|------|-----|-------|-----|
| (i)  | 51  | (ii) | 51  | (iii) | 101 |
| (iv) | 101 | (v)  | 261 | (vi)  | 261 |

#### Answer 5.

- |      |    |      |     |       |     |
|------|----|------|-----|-------|-----|
| (i)  | 16 | (ii) | 58  | (iii) | 457 |
| (iv) | 6  | (v)  | 238 | (vi)  | 28  |

#### Answer 6.

- |      |      |      |     |       |     |
|------|------|------|-----|-------|-----|
| (i)  | 14   | (ii) | 39  | (iii) | -1  |
| (iv) | 1110 | (v)  | 155 | (vi)  | -21 |

#### Answer 7.

- |      |    |      |     |       |    |
|------|----|------|-----|-------|----|
| (i)  | 4  | (ii) | 3   | (iii) | 1  |
| (iv) | -6 | (v)  | 0.5 | (vi)  | 48 |

Domain is all real numbers except zero (i.e.  $x \neq 0$ ) because otherwise you could try to divide by zero which has no meaning in mathematics. It's undefined.

A division by zero error caused a car factory robot to kill a worker in 1994.

#### Answer 8.

- |      |    |      |   |       |   |
|------|----|------|---|-------|---|
| (i)  | 3  | (ii) | 6 | (iii) | 9 |
| (iv) | 12 | (v)  | 1 | (vi)  | 5 |

Restriction on the domain,  $x \geq 0$ , is to avoid trying to take  $\sqrt{\quad}$  of a negative number.

#### Answer 9.

- |     |    |      |                        |       |   |
|-----|----|------|------------------------|-------|---|
| (i) | 68 | (ii) | $\frac{5}{8}$ or 0.625 | (iii) | $\frac{1}{1000} + \frac{1}{10} = 0.101$ |
|-----|----|------|------------------------|-------|---|

**Answer 10.**

(i) 5                      (ii) 9                      (iii)  $\frac{1}{2}$  or 0.5

Restriction on the domain,  $x \geq 0$ , is to avoid trying to take  $\sqrt{\quad}$  of a negative number.

**Answer 11.**

(i)  $\frac{1}{4}$                       (ii)  $\frac{1}{25}$                       (iii)  $\frac{1}{16}$

Restriction on the domain,  $x \neq 0$ , is to avoid trying to divide by zero.

**Answer 12.**

(i) 12                      (ii) 42                      (iii) 132

Restriction on the domain,  $x \geq 0$ , is to avoid trying to take  $\sqrt{\quad}$  of a negative number.

**Answer 13.**

(i)  $1\frac{1}{3}$                       (ii)  $1\frac{1}{10}$                       (iii)  $\frac{9}{10}$   
(iv)  $1\frac{1}{5}$                       (v)  $\frac{4}{5}$                       (vi)  $\frac{99}{100}$

**Answer 14.**

(i)  $z^2 + 2z + 1$                       (ii)  $z^2 + 10z + 25$                       (iii)  $4z^2 + 12z + 9$

**Answer 15.**

(i) 4                      (ii) -2  
(iii)  $w^2 + 1$                       (iv)  $w^2 + w + 1$

**Answer 16.**

(i) 20                      (ii) 12  
(iii)  $z^2 + 3z + 2$                       (iv)  $z^4 + 2z^3 + 2z^2 + z$



## Chapter 2

### Functions : GCSE

#### 2.1 Functions applied to algebra, rather than only numbers

Previously we looked at what the following function did to various numbers;

$$f(x) = x^2 + 1$$

In words, whatever the input, square it and then add 1.

For example,

$$\begin{aligned} f(5) &= 5^2 + 1 \\ &= 26 \end{aligned}$$

We can also drop algebraic expressions into function  $f$ .

The algebra dropped in will be squared and then have 1 added on.

For example, let's drop  $4z$  into function  $f$ .

$$\begin{aligned} f(4z) &= (4z)^2 + 1 \\ &= (4z)(4z) + 1 \\ &= 16z^2 + 1 \end{aligned}$$

What will happen if  $3z + 1$  is dropped into function  $f$  ?

Write the answer without any brackets.

## 2.2 You try

Here are five questions for you to try.

The answers are on the following page - Don't look yet !

*Try each one yourself first, then check over the page to see if you got it correct.*

If  $h(x) = 4x + 13$ , find expressions that do not involve brackets for;

**Try 1.**             $h(5)$

**Try 2.**             $h(3z)$

**Try 3.**             $h(3z + 1)$

**Try 4.**             $h(z^2 + 7)$

**Try 5.**             $h(x^2 + 7)$

**Reminder :**

$$h(x) = 4x + 13$$

In words: Multiply the input by four and then add on 13.

**So the answers are :**

**Try 1.**             $h(5)$

$$\begin{aligned} h(5) &= 4 \times 5 + 13 \\ &= 33 \end{aligned}$$

**Try 2.**             $h(3z)$

$$\begin{aligned} h(3z) &= 4 \times 3z + 13 \\ &= 12z + 13 \end{aligned}$$

**Try 3.**             $h(3z + 1)$

$$\begin{aligned} h(3z + 1) &= 4(3z + 1) + 13 \\ &= 12z + 4 + 13 \\ &= 12z + 17 \end{aligned}$$

**Try 4.**             $h(z^2 + 7)$

$$\begin{aligned} h(z^2 + 7) &= 4(z^2 + 7) + 13 \\ &= 4z^2 + 28 + 13 \\ &= 4z^2 + 41 \end{aligned}$$

**Try 5.**             $h(x^2 + 7)$

$$\begin{aligned} h(x^2 + 7) &= 4(x^2 + 7) + 13 \\ &= 4x^2 + 28 + 13 \\ &= 4x^2 + 41 \end{aligned}$$

## 2.3 Exercise

### Question 1.

If  $f(x) = 5x + 11$ , find expressions that do not involve brackets for;

(i)  $f(3)$                       (ii)  $f(5)$                       (iii)  $f(12)$

(iv)  $f(2z)$

(v)  $f(6z + 5)$

(vi)  $f(3z + 7)$

(vii)  $f(-2)$                       (viii)  $f(-1)$                       (ix)  $f(0.1)$

(x)  $f(4z - 2)$

(xi)  $f(8z - 3)$

**Question 2.**

If  $g(x) = 8x - 10$ , find expressions that do not involve brackets for;

(i)  $g(2)$                       (ii)  $g(6)$                       (iii)  $g(0)$

(iv)  $g(3z)$

(v)  $g(5z + 2)$

(vi)  $g(7 + 2z)$

(vii)  $g(-2)$                       (viii)  $g(-5)$                       (ix)  $g(0.1)$

(x)  $g(-4z)$

(xi)  $g(8z - 1)$

**Question 3.**

If  $h(x) = x^2$ , find expressions that do not involve brackets for;

(i)  $h(3)$                       (ii)  $h(-3)$                       (iii)  $h(8)$

(iv)  $h(3z)$

(v)  $h(5z + 1)$

**HINT : FOIL**

(vi)  $h(2z + 7)$

(vii)  $h\left(\frac{3}{2}\right)$                       (viii)  $h\left(-\frac{3}{10}\right)$                       (ix)  $h(0.5)$

(x)  $h(-4z)$

(xi)  $h(6z - 2)$

**Question 4.**

If  $k(x) = 7x + 3$ , find expressions that do not involve brackets for;

(i)  $k(7)$                       (ii)  $k(6)$                       (iii)  $k(-11)$

(iv)  $k(3x)$

(v)  $k(2x + 10)$

(vi)  $k(5x + 1)$

(vii)  $k(0.1)$                       (viii)  $k(-0.1)$                       (ix)  $k\left(\frac{1}{2}\right)$

(x)  $k(4x - 1)$

(xi)  $k(7x - 2)$

**Question 5.**

If  $m(x) = x^2 + 5$ , find expressions that do not involve brackets for;

(i)  $m(7)$                       (ii)  $m(-7)$                       (iii)  $m(20)$

(iv)  $m(3x)$

(v)  $m(-3x)$

(vi)  $m(5x + 1)$

**HINT : FOIL**

(vii)  $m(-1)$                       (viii)  $m\left(\frac{1}{2}\right)$                       (ix)  $m\left(\frac{3}{2}\right)$

(x)  $m(4x + 7)$

(xi)  $m(7x - 2)$



**Question 6.**

If  $n(x) = 4x^2 + 1$ , find expressions that do not involve brackets for;

(i)  $n(3)$                       (ii)  $n(-3)$                       (iii)  $n(100)$

(iv)  $n(3x)$

(v)  $n(-3x)$

(vi)  $n(7x + 1)$

**HINT : FOIL**

(vii)  $n(0)$                       (viii)  $n\left(\frac{1}{2}\right)$                       (ix)  $n\left(\frac{3}{2}\right)$

(x)  $n(2x + 3)$

(xi)  $n(5x - 2)$

## 2.4 Homework

*These questions are harder.*

*If you get stuck on one, miss it out and come back to it later.*

### Question 1.

If  $f(x) = 8x + 5$ , find;

- (i)  $f(4)$                       (ii)  $f(-11)$                       (iii)  $f(0.25)$

### Question 2.

If  $g(x) = 13 - x$ , find;

- (i)  $g(7)$                       (ii)  $g(25)$                       (iii)  $g(-5)$

### Question 3.

If  $h(x) = \sqrt{25 - x^2}$ ,  $-5 \leq x \leq 5$ . find;

- (i)  $h(4)$                       (ii)  $h(0)$                       (iii)  $h(-5)$

- (iv) Try to find  $h(7)$

Hence explain the restriction on the domain that  $-5 \leq x \leq 5$

### Question 4.

If  $k(x) = 8x^2 + 6x + 13$ , find;

- (i)  $k(5)$                       (ii)  $k(-1)$                       (iii)  $k(0.5)$

**Question 5.**

If  $m(x) = x^3 + x^2$ , find;

- (i)  $m(3)$                       (ii)  $m(-1)$                       (iii)  $m(0.5)$

**Question 6.**

If  $n(x) = \frac{12}{x}$ ,  $x \neq 0$ , find;

- (i)  $n(6)$                       (ii)  $n(0.25)$                       (iii)  $n(36)$

- (iv) Try to find  $n(0)$  and hence explain the restriction on the domain that  $x \neq 0$ .

**Question 7.**

If  $f(x) = 3x^2$ , find;

- (i)  $f(11)$                       (ii)  $f(w+1)$                       (iii)  $f(2x)$

*Write your answers without any brackets.*

**Question 8.**

If  $f(x) = x^3 + x$ , find;

- (i)  $f(4)$                       (ii)  $f(3w)$                       (iii)  $f(x^2)$

*Write your answers without any brackets.*

**Question 9.**

If  $f(x) = x^2 + 1$ ; find;

(i)  $f(14)$

(ii)  $f(w + 4)$

(iii)  $f(3\sqrt{x})$

*Write your answers without any brackets.*

**Question 10.**

If  $f(x) = x^3$  find  $f(2x + 1)$ .

*Express your answer without using any brackets.*

## 2.5 Answers

### 2.5.1 Solutions (Introduction)

$$\begin{aligned}f(x) &= x^2 + 1 \\f(3z + 1) &= (3z + 1)^2 + 1 \\&= (3z + 1)(3z + 1) + 1 \\&= 9z^2 + 6z + 1 + 1 \\&= 9z^2 + 6z + 2\end{aligned}$$

### 2.5.2 Solutions (Exercise)

#### Answer 1.

- |       |            |        |    |       |      |
|-------|------------|--------|----|-------|------|
| (i)   | 26         | (ii)   | 36 | (iii) | 71   |
| (iv)  | $10z + 11$ |        |    |       |      |
| (v)   | $30z + 36$ |        |    |       |      |
| (vi)  | $15z + 46$ |        |    |       |      |
| (vii) | 1          | (viii) | 6  | (ix)  | 11.5 |
| (x)   | $20z + 1$  |        |    |       |      |
| (xi)  | $40z - 4$  |        |    |       |      |

#### Answer 2.

- |       |             |        |     |       |      |
|-------|-------------|--------|-----|-------|------|
| (i)   | 6           | (ii)   | 38  | (iii) | -10  |
| (iv)  | $24z - 10$  |        |     |       |      |
| (v)   | $40z + 6$   |        |     |       |      |
| (vi)  | $46 + 16z$  |        |     |       |      |
| (vii) | -26         | (viii) | -50 | (ix)  | -9.2 |
| (x)   | $-32z - 10$ |        |     |       |      |
| (xi)  | $64z - 18$  |        |     |       |      |

#### Answer 3.

- |       |                      |        |                        |       |                      |
|-------|----------------------|--------|------------------------|-------|----------------------|
| (i)   | 9                    | (ii)   | 9                      | (iii) | 64                   |
| (iv)  | $9z^2$               |        |                        |       |                      |
| (v)   | $25z^2 + 10z + 1$    |        |                        |       |                      |
| (vi)  | $4z^2 + 28z + 49$    |        |                        |       |                      |
| (vii) | $\frac{9}{4} = 2.25$ | (viii) | $\frac{9}{100} = 0.09$ | (ix)  | $\frac{1}{4} = 0.25$ |
| (x)   | $16z^2$              |        |                        |       |                      |
| (xi)  | $36z^2 - 24z + 4$    |        |                        |       |                      |

**Answer 4.**

- |       |            |        |     |       |      |
|-------|------------|--------|-----|-------|------|
| (i)   | 52         | (ii)   | 45  | (iii) | - 74 |
| (iv)  | $21x + 3$  |        |     |       |      |
| (v)   | $14x + 73$ |        |     |       |      |
| (vi)  | $35x + 10$ |        |     |       |      |
| (vii) | 3.7        | (viii) | 2.3 | (ix)  | 6.5  |
| (x)   | $28x - 4$  |        |     |       |      |
| (xi)  | $49x - 11$ |        |     |       |      |

**Answer 5.**

- |       |                    |        |      |       |      |
|-------|--------------------|--------|------|-------|------|
| (i)   | 54                 | (ii)   | 54   | (iii) | 405  |
| (iv)  | $9x^2 + 5$         |        |      |       |      |
| (v)   | $9x^2 + 5$         |        |      |       |      |
| (vi)  | $25x^2 + 10x + 6$  |        |      |       |      |
| (vii) | 6                  | (viii) | 5.25 | (ix)  | 7.25 |
| (x)   | $16x^2 + 56x + 54$ |        |      |       |      |
| (xi)  | $49x^2 - 28x + 9$  |        |      |       |      |

**Answer 6.**

- |       |                     |        |    |       |        |
|-------|---------------------|--------|----|-------|--------|
| (i)   | 37                  | (ii)   | 37 | (iii) | 40 001 |
| (iv)  | $36x^2 + 1$         |        |    |       |        |
| (v)   | $36x^2 + 1$         |        |    |       |        |
| (vi)  | $196x^2 + 56x + 5$  |        |    |       |        |
| (vii) | 1                   | (viii) | 2  | (ix)  | 10     |
| (x)   | $16x^2 + 48x + 37$  |        |    |       |        |
| (xi)  | $100x^2 - 80x + 17$ |        |    |       |        |

### 2.4.3 Solutions (2.4 Homework)

**Answer 1.**

(i) 37                      (ii) - 83                      (iii) 7

**Answer 2.**

(i) 6                      (ii) - 12                      (iii) 18

**Answer 3.**

(i) 3                      (ii) 5                      (iii) 0  
(iv) The square root of a negative number is not a real number

**Answer 4.**

(i) 243                      (ii) 15                      (iii) 18

**Answer 5.**

(i) 36                      (ii) 0                      (iii)  $\frac{3}{8}$

**Answer 6.**

(i) 2                      (ii) 48                      (iii)  $\frac{6}{13}$   
(iv) Division by zero is not defined

**Answer 7.**

(i) 363                      (ii)  $3w^2 + 6w + 3$                       (iii)  $12x^2$

**Answer 8.**

(i) 68                      (ii)  $27w^3 + 3w$                       (iii)  $x^6 + x^2$

**Answer 9.**

(i) 197                      (ii)  $w^2 + 8w + 17$                       (iii)  $9x + 1$

**Answer 10.**

$$\begin{aligned}(2x + 1)^3 &= 1 \times (2x)^0 \times (1)^3 \\ &+ 3 \times (2x)^1 \times (1)^2 \\ &+ 3 \times (2x)^2 \times (1)^1 \\ &+ 1 \times (2x)^3 \times (1)^0 \\ &= 1 + 6x + 12x^2 + 8x^3\end{aligned}$$

## Chapter 3

## Functions : GCSE

### 3.1 Composite Functions ( Numerical Questions )

We have got used to the idea of substituting a number into a function, and the function 'doing stuff' to the number.

Typically, a different number then comes out.

We're now going to take that output number and feed it back around to the input again.

Alternatively we could use it as the input to another, completely different, function.

### 3.2 A numerical example

Let  $p$  and  $q$  be the functions;

$$p(x) = 8x + 5$$

$$q(x) = 3x + 11$$

Evaluate each of the following;

(i)  $p(6)$

(ii)  $q(6)$

(iii)  $p(-2)$

(iv)  $p(0.25)$

(v)  $qq(3)$

(vi)  $pp(0)$

**DANGER**

(vii)  $pq(-1)$

**DANGER**

(viii)  $pq(0)$

**DANGER**

(ix)  $qp(-0.5)$



### 3.3 Exercise

#### Question 1

Let  $f$  and  $g$  be the functions;

$$f(x) = 5x - 12$$

$$g(x) = 4x + 3$$

Evaluate each of the following;

(i)  $f(3)$

(ii)  $g(6)$

(iii)  $f(10)$

(iv)  $g(16)$

(v)  $ff(4)$

(vi)  $gg(5)$

(vii)  $fg(1)$

(viii)  $fg(0)$

(ix)  $gf(0)$

#### Question 2

Let  $m$  and  $n$  be the functions;

$$m(x) = x^2 - 75$$

$$n(x) = \frac{4x}{5}$$

Evaluate each of the following;

(i)  $m(1)$

(ii)  $n(15)$

(iii)  $m(-1)$

(iv)  $n(-15)$

(v)  $mm(9)$

(vi)  $nn(50)$

(vii)  $mn(0)$

(viii)  $mn(10)$

(ix)  $mnm(10)$

**Question 3.**

Let  $s$  and  $t$  be the functions;

$$s(x) = 6x + 1$$

$$t(x) = (x + 1)^2$$

Evaluate each of the following;

(i)  $s(8)$

(ii)  $t(6)$

(iii)  $s(9)$

(iv)  $t(-4)$

(v)  $ss(1)$

(vi)  $tt(2)$

(vii)  $st(9)$

(viii)  $st(2)$

(ix)  $ts(-1)$

**Question 4.**

Let two functions,  $m$  and  $n$ , be;

$$m(x) = 10x - 8$$

$$n(x) = 100 - x^2$$

Find each of the following;

(i)  $m(3)$

(ii)  $n(7)$

(iii)  $n(-6)$

(iv)  $mn(8)$

(v)  $mm(4)$

(vi)  $nm(2)$

**Question 5.**

Let two functions,  $s$  and  $t$ , be;

$$s(x) = x^2 + x$$

$$t(x) = 3x - 2$$

Find each of the following;

(i)  $s(5)$                       (ii)  $s s s(1)$                       (iii)  $t t t(1)$

(iv)  $t s(10)$                       (v)  $s t(3)$                       (vi)  $t s(-1)$

**Question 6**

If  $f(x) = \sqrt{2x + 11}$ ,  $x \geq -\frac{11}{2}$  and  $g(x) = 5x$ , what is;

(i)  $f(7)$                       (ii)  $g f(19)$                       (iii)  $f g(11)$

(iv)  $f g(-0.2)$                       (v)  $g f(-\frac{11}{2})$                       (vi)  $f g(x)$

### 3.4 Homework

#### Question 1.

Let  $f(x) = \frac{x^3}{x^2 + 6x}$ , with domain  $x$  such that  $x \neq 0$ ,  $x \neq -6$ .

Find;

(i)  $f(2)$

(ii)  $f(-1)$

#### Question 2

Let  $g(x) = \sqrt{x^2 + 5x}$ , with domain  $x$  such that  $x \leq -5$  or  $x \geq 0$ .

Find;

(i)  $g(4)$

(ii)  $g(-9)$

#### Question 3.

Let two functions  $f$  and  $g$  be;

$$f(x) = \frac{16}{x}, \quad x \neq 0$$

and  $g(x) = x + 2,$

**Remember :**  $gf(16)$  means put 16 into function  $f$  first, then  $g$ .

Find;

(i)  $gf(16)$

(ii)  $fg(2)$

(iii)  $gfg(2)$

(iv)  $ffg(14)$

(v)  $gfg(0)$

(vi)  $fgf(8)$

**Question 4.**

Let two functions,  $e$  and  $z$ , be;

$$e(x) = (x + 3)^2$$

and  $z(x) = 7x - 4$

Find each of the following;

(i)  $e(-5)$       (ii)  $e z(2)$       (iii)  $z z(-1)$

(iv)  $z e e(-2)$       (v)  $z z(x)$       (vi)  $e z(x)$

**Question 5.**

Let two functions,  $f$  and  $g$  be;

$$f(x) = x^2$$

and  $g(x) = 50 - x$

Work out;

(i)  $f(-5)$       (ii)  $g(-5)$       (iii)  $f g(10)$

(iv)  $g f(10)$       (v)  $f f f(2)$       (vi)  $g g g g(7)$

(vii) Find  $f g(x)$  and write your answer without brackets.

(viii) By trial and improvement find a value of  $x$  such that  $f(x) = g(x)$ .

### 3.5 Answers

#### 3.5.1 Solutions (3.1 Introduction)

(i)	53	(ii)	29	(iii)	- 11
(iv)	7	(v)	71	(vi)	45
(vii)	69	(viii)	93	(ix)	14

#### 3.5.2 Solutions (3.3 Exercise)

##### Answer 1.

(i)	3	(ii)	27	(iii)	38
(iv)	67	(v)	28	(vi)	95
(vii)	23	(viii)	3	(ix)	- 45

##### Answer 2.

(i)	- 74	(ii)	12	(iii)	- 74
(iv)	- 12	(v)	- 39	(vi)	32
(vii)	- 75	(viii)	- 11	(ix)	325

##### Answer 3.

(i)	49	(ii)	49	(iii)	55
(iv)	9	(v)	43	(vi)	100
(vii)	601	(viii)	55	(ix)	16

##### Answer 4.

(i)	22	(ii)	51	(iii)	64
(iv)	352	(v)	312	(vi)	- 44

##### Answer 5.

(i)	30	(ii)	42	(iii)	1
(iv)	328	(v)	56	(vi)	- 2

##### Answer 6.

(i)	5	(ii)	35	(iii)	11
(iv)	3	(v)	0	(vi)	$\sqrt{10x + 11}$

### 3.5.3 Solutions (3.4 Homework)

#### Answer 1.

(i)  $\frac{1}{2}$                       (ii)  $\frac{1}{5}$

#### Answer 2.

(i) 6                              (ii) 6

#### Answer 3.

(i) 3                              (ii) 4                              (iii) 12  
(iv) 16                              (v) 10                              (vi) 4

#### Answer 4.

(i) 4                              (ii) 169                              (iii) - 81  
(iv) 108                              (v)  $49x - 32$                               (vi)  $49x^2 - 14x + 1$

#### Answer 5.

(i) 25                              (ii) 55                              (iii) 1600  
(iv) - 50                              (v) 256                              (vi) 7  
(vii)  $2500 - 100x + x^2$   
(viii)

*The exact answer will be given by*

$$\begin{aligned}x^2 &= 50 - x \\x^2 + x - 50 &= 0 \\x &= \frac{-1 \pm \sqrt{201}}{2} \\x &= 6.5887, 7.5887\end{aligned}$$

**Finding a the input, given the output**

We are now proficient at working out what happens to a number when it is used as an input to a function.

**4.1 Example of previous problem**

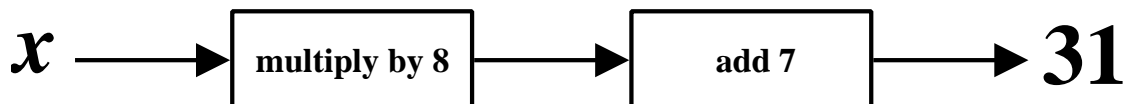
If  $f(x) = \frac{x^2 + 5}{\sqrt{x + 15}}$ ,  $x \geq -15$ , find  $f(1)$ .

**4.2 Example of today's problem**

Sometimes the output is known, and the input sought.

If  $f(x) = 8x + 7$ , find  $x$  such that  $f(x) = 31$ .

The function  $f(x) = 8x + 7$  can be illustrated by this flow chart.



Work out  $x$  by tracing a path backward through the diagram, undoing the various action boxes. What is the input,  $x$ ?

Algebraically, to save drawing flow charts, you need to solve the equation  $8x + 7 = 31$ .

Here is a second example to try.

Try it yourself, then check your answer with mine which is over the page.

If  $f(x) = 6x + 11$ , find  $x$  such that  $f(x) = 53$ .



Solve,  $6x + 11 = 53$

$$6x = 53 - 11$$

$$6x = 42$$

$$x = \frac{42}{6}$$

$$x = 7$$

### 4.3 Exercise

#### Question 1.

If  $f(x) = 4x + 13$ , find  $x$  such that  $f(x) = 41$ .

#### Question 2.

If  $g(x) = 18 + 5x$ , find  $x$  such that  $g(x) = 103$ .

#### Question 3.

If  $h(x) = 3(8x - 3)$ , find  $x$  such that  $h(x) = 135$ .

#### Question 4.

If  $k(x) = \frac{5x}{4} + 7$ , find  $x$  such that  $k(x) = 37$ .

#### Question 5.

If  $m(x) = \frac{3x}{5} - 1$ , find  $x$  such that  $m(x) = 32$ .

**Question 6.**

If  $n(x) = 68 - 7x$ , find  $x$  such that  $n(x) = 26$ .

**Question 7.**

If  $p(x) = 6(2x + 3)$ , find  $x$  such that  $p(x) = 48$ .

**Question 8.**

If  $q(x) = 6(13 - x)$ , find  $x$  such that  $q(x) = 72$ .

**Question 9.**

If  $r(x) = \frac{x + 17}{3}$ , find  $x$  such that  $r(x) = 40$ .

**Question 10.**

If  $s(x) = \frac{3x + 2}{5}$ , find  $x$  such that  $s(x) = 10$ .

**Question 11.**

If  $t(x) = 5x^3$ , find  $x$  such that  $t(x) = 320$ .

**Question 12.**

If  $u(x) = \frac{21-x}{4}$ , find  $x$  such that  $u(x) = 6$ .

**Question 13.**

If  $v(x) = 14\sqrt{x} + 111$ ,  $x \geq 0$ , find  $x$  such that  $v(x) = 181$ .

**Question 14.**

If  $w(x) = \frac{3x-5}{2-x}$ ,  $x \neq 2$ , find  $x$  such that  $w(x) = 7$ .

**Question 15.**

If  $a(x) = \frac{\sqrt{x}+7}{2}$ ,  $x \geq 0$ , find  $x$  such that  $a(x) = 16$ .

**Question 16.**

If  $b(x) = \frac{x+2}{3} + 9$ , find  $x$  such that  $b(x) = 14$ .

**Question 17.**

If  $c(x) = \frac{105}{x}$ ,  $x \neq 0$ , find  $x$  such that  $c(x) = 7$ .

**Question 18.**

If  $d(x) = \frac{5x+1}{x+1}$ ,  $x \neq -1$ , find  $x$  such that  $d(x) = 4$ .

**Question 19.**

If  $e(x) = \sqrt[3]{x} + 7$ , find  $x$  such that  $e(x) = 11$ .

**Question 20.**

If  $f(x) = \frac{5x-4}{x+1}$ ,  $x \neq -1$ , find  $x$  such that  $f(x) = 8$ .

**Question 21.**

If  $g(x) = \frac{2}{x} + \frac{3}{x}$ ,  $x \neq 0$ , find  $x$  such that  $g(x) = 20$ .

**Question 22.**

If  $h(x) = \sqrt{13 + 3x}$ ,  $x \geq -\frac{13}{3}$ , find  $x$  such that  $h(x) = 7$ .

**Question 23.**

If  $k(x) = 29 - 24\sqrt{x}$ ,  $x \geq 0$ , find  $x$  such that  $k(x) = 17$ .

**Question 24.**

If  $p(x) = x^2 + x$ , find  $x$  such that  $p(x) = 1$ .

**NOTE:** The equation  $ax^2 + bx + c = 0$ , where  $a, b$  and  $c$  are constants, has the solutions given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Question 25.**

If  $m(x) = \frac{3}{x} + \frac{5}{2x}$ ,  $x \geq 0$ , find  $x$  such that  $m(x) = 33$ .

**Question 26.**

If  $n(x) = \frac{1}{3x} + \frac{3}{5x}$ ,  $x \neq 0$ , find  $x$  such that  $n(x) = 7$ .

## Chapter 5

## Functions : GCSE

### 5.1 Composite Functions ( Algebra Questions )

We start with two functions,  $f$  and  $g$  given by;

$$f(x) = 5x - 2$$

$$g(x) = 4x + 1$$

Previously, we worked these with numbers.

For example, what is  $fg(2)$  ?



Now we make the jump to doing the same sort of thing in algebra.

Work out  $fg(2z)$ .



Now try to work out  $fg(3z + 1)$



My answer is over the page.  
Turn over to see if you concur.

$$\begin{aligned}
fg(3z + 1) &= f(4(3z + 1) + 1) && \text{because } g(x) = 4x + 1 \\
&= f(12z + 4 + 1) \\
&= f(12z + 5) \\
&= 5(12z + 5) - 2 && \text{because } f(x) = 5x - 2 \\
&= 60z + 25 - 2 \\
&= 60z + 23
\end{aligned}$$

## 5.2 Exercise

### Question 1.

Let  $p$  and  $q$  be the functions;

$$p(x) = 3x + 7$$

$$q(x) = 10x + 9$$

Evaluate each of the following;

(i)  $pq(0)$

(ii)  $pq(1)$

(iii)  $pq(5z)$

(iv)  $pq(2z + 3)$

(v)  $qq(-1)$

(vi)  $ppp(-3)$



**Question 2.**

Let  $h$  and  $d$  be the functions;

$$h(x) = 3x + 7$$

$$d(x) = 5x + 9$$

Evaluate each of the following;

(i)  $hd(4)$

(ii)  $hd(-1)$

(iii)  $hd(4z)$

(iv)  $hd(3z+3)$

**Question 3.**

Let  $f$  and  $g$  be the functions;

$$f(x) = 6x - 5$$

$$g(x) = 2x + 11$$

Evaluate each of the following;

(i)  $gf(0)$

(ii)  $gf(2)$

(iii)  $gf(3z)$

(iv)  $gf(5z+1)$

**Question 4.**

$$f(x) = x + 3$$

$$g(x) = 4x$$

Find  $fg(x)$

**Question 5.**

$$f(x) = 3x + 2$$

$$g(x) = 5x - 4$$

Find an expression for  $gf(x)$  that does not contain any brackets.

**Question 6.**

$$f(x) = 7x - 5$$

$$g(x) = 10 - x$$

Find an expression for  $fg(x)$  that does not contain any brackets.

**Question 7.**

$$f(x) = x^2 + 5$$

$$g(x) = 2x - 1$$

Find an expression for  $fg(x)$  that does not contain any brackets.

**Question 8.**

$$f(x) = 7x - 6$$

$$g(x) = \frac{3}{x + 4}$$

(i) Find an expression for  $fg(x)$  that does not contain any brackets.

(ii) Find an expression for  $gf(x)$  that does not contain any brackets.

**Question 9.**

$$f(x) = 5x + 2$$

$$g(x) = 4x - 9$$

(i) Calculate  $fg(3)$

(ii) Find an expression for  $fg(x)$  that does not contain any brackets.

(iii) Now use your part (ii) formula to calculate  $fg(3)$  again.  
Check it agrees with your answer to part (i)!

**Question 10.**

$$f(x) = x^2 - 6$$

$$g(x) = 3x + 1$$

(i) Find an expression for  $fg(x)$  that does not contain any brackets.

(ii) Hence solve the equation  $fg(x) = 43$

**Question 11.**

$$f(x) = 2x^2 + 3x - 4$$

$$g(x) = x - 5$$

(i) Show carefully that  $fg(x) = 2x^2 - 17x + 31$

(ii) Hence solve the equation  $fg(x) = 10 - 4x$

**Question 12.**

Let  $u$  and  $v$  be the functions;

$$u(x) = x^2 - 5$$

$$v(x) = 5x + 2$$

Evaluate each of the following;

(i)  $v u (3)$

(ii)  $v u (2)$

(iii)  $v u (5z)$

(iv)  $v u (2z + 1)$  **HINT : FOIL**

**Question 13.**

Let  $m$  and  $n$  be the functions;

$$m(x) = 9x - 5$$

$$n(x) = \sqrt{x - 7}, \quad x \geq 7,$$

Evaluate each of the following;

(i)  $m n (8)$

(ii)  $m n (56)$

(iii)  $m n (z^2 + 7)$

(iv)  $m n (4z^2 + 7)$

**Question 14.**

$$f(x) = 3x + 7$$

$$g(x) = \frac{5}{x - 2}$$

(i) Find an expression for  $fg(x)$  that does not contain any brackets.

(ii) Find an expression for  $gf(x)$  that does not contain any brackets.

**Question 15.**

$$f(x) = 4x + 5$$

$$g(x) = 3x - 7$$

(i) Calculate  $fg(2)$

(ii) Find an expression for  $fg(x)$  that does not contain any brackets.

(iii) Now use your part (ii) formula to calculate  $fg(2)$  again.  
Check it agrees with your answer to part (i)!

**Question 16.**

$$f(x) = x^2 + 8$$

$$g(x) = 2x - 5$$

(i) Find an expression for  $fg(x)$  that does not contain any brackets.

(ii) Hence solve the equation  $fg(x) = 17$

**Question 17.**

$$f(x) = 3x^2 - 2x + 5$$

$$g(x) = x + 4$$

(i) Show carefully that  $fg(x) = 3x^2 + 22x + 45$

(ii) Hence solve the equation  $fg(x) = 27x + 43$

**Question 18.**

Let  $f$  and  $g$  be the functions;

$$f(x) = 7 - 6x$$

$$g(x) = 5x + 7$$

Evaluate each of the following;

(i)  $fg(-1)$

(ii)  $fg(-0.2)$

(iii)  $fg(4z + 1)$

(iv)  $fg(1 - z^2)$



## 5.3 Answers

### 5.3.1 Solutions (Introductory Example)

☞  $fg(2) = 43$

☞  $fg(2z) = f(8z + 1)$   
 $= 40z + 3$

☞  $fg(3z + 1) = f(12z + 5)$   
 $= 60z + 23$

### 5.3.2 Solutions (5.2 Exercise)

#### Answer 1.

(i) 34

(iii)  $150z + 34$

(v) -1

(ii) 64

(iv)  $60z + 124$

(vi) 10

#### Answer 2.

(i) 94

(iii)  $60z + 34$

(ii) 19

(iv)  $45z + 79$

#### Answer 3.

(i) 1

(iii)  $36z + 1$

(ii) 25

(iv)  $60z + 13$

#### Answer 4.

$4x + 3$

#### Answer 5.

$15x + 6$

#### Answer 6.

$65 - 7x$

#### Answer 7.

$4x^2 - 4x + 6$

#### Answer 8.

(i)  $\frac{21}{x+4} - 6$

(ii)  $\frac{3}{7x-2}$

**Answer 9.**

- (i) 17
- (ii)  $20x - 43$
- (iii) 17 again

**Answer 10.**

- (i)  $9x^2 + 6x - 5$
- (ii)  $2, -\frac{8}{3}$

**Answer 11.**

- (ii) 3.5, 3

**Answer 12.**

- (i) 22
- (ii) -3
- (iii)  $125z^2 - 23$
- (iv)  $20z^2 + 20z - 18$

**Answer 13.**

- (i) 4
- (ii) 58
- (iii)  $9z - 5$
- (iv)  $18z - 5$

**Answer 14.**

- (i)  $\frac{15}{x-2} + 7$
- (ii)  $\frac{5}{3x+5}$

**Answer 15.**

- (i) 1
- (ii)  $12x - 23$
- (iii) 1 again

**Answer 16.**

- (i)  $4x^2 + 20x + 33$
- (ii) -1, -4

**Answer 17.**

- (ii)  $1, \frac{2}{3}$

**Answer 18.**

- (i) -5
- (ii) -29
- (iii)  $-120z - 65$
- (iv)  $30z^2 - 65$

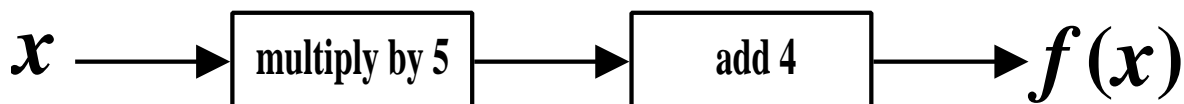
6.1 Inverse Functions from Flow Diagrams

Study the flow diagram below.

We're going to write down two algebraic expressions suggested by this flow diagram.

Firstly, what function does the flow diagram represent ?

Secondly, if you were to go backwards through the flow diagram, what *inverse function* would be obtained ?



$$f(x) =$$



$$f^{-1}(x) =$$



Having extracted the algebra we need from the flow diagram we can now answer some easy questions.

Determine the value of,

(i)  $f(8)$



(ii)  $f^{-1}(29)$



(iii)  $f(11)$



(iv)  $f^{-1}(44)$

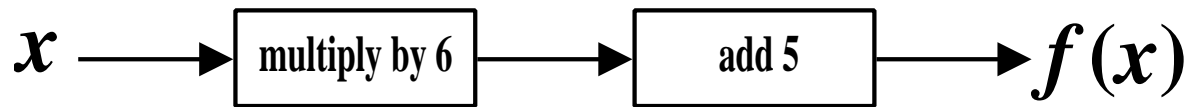


Explain the connection between (i) and (iv).



## 6.2 Exercise

### Question 1.



(a) Write down (i)  $f(x)$

(ii)  $f^{-1}(x)$

(b) Determine the value of,

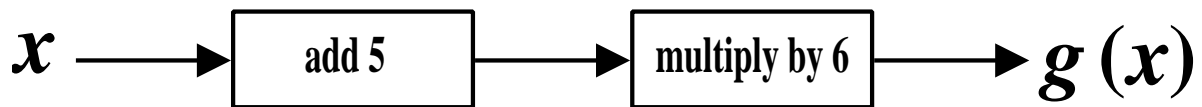
(i)  $f(4)$

(ii)  $f^{-1}(41)$

(iii)  $f(9)$

(iv)  $f^{-1}(71)$

### Question 2.



(a) Write down (i)  $g(x)$

(ii)  $g^{-1}(x)$

(b) Determine the value of,

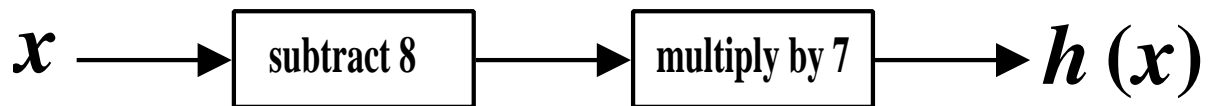
(i)  $g(4)$

(ii)  $g^{-1}(48)$

(iii)  $g(15)$

(iv)  $g^{-1}(72)$

**Question 3.**



(a) Write down (i)  $h(x)$

(ii)  $h^{-1}(x)$

(b) Determine the value of,

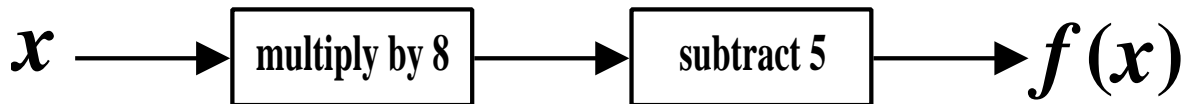
(i)  $h(13)$

(ii)  $h^{-1}(77)$

(iii)  $h(28)$

(iv)  $h^{-1}(490)$

**Question 4.**



(a) Write down (i)  $f(x)$

(ii)  $f^{-1}(x)$

(b) Determine the value of,

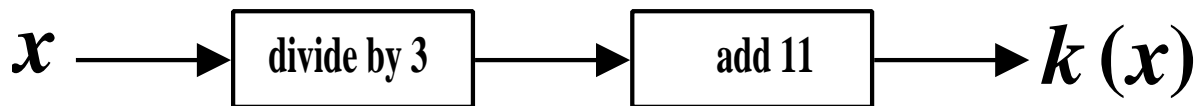
(i)  $f(6)$

(ii)  $f^{-1}(19)$

(iii)  $f(1.5)$

(iv)  $f^{-1}(83)$

**Question 5.**



(a) Write down (i)  $k(x)$

(ii)  $k^{-1}(x)$

(b) Determine the value of,

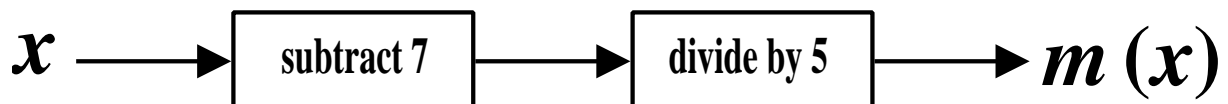
(i)  $k(66)$

(ii)  $k^{-1}(15)$

(iii)  $k(21)$

(iv)  $k^{-1}(19)$

**Question 6.**



(a) Write down (i)  $m(x)$

(ii)  $m^{-1}(x)$

(b) Determine the value of,

(i)  $m(22)$

(ii)  $m^{-1}(1)$

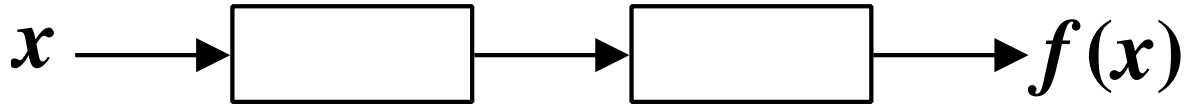
(iii)  $m(37)$

(iv)  $m^{-1}(8)$

**Question 7.**

- (a) Fill in the flow diagram for the function

$$f(x) = 6x - 7$$



- (b) Write down  $f^{-1}(x)$

- (c) Determine the value of,

(i)  $f(10)$

(ii)  $f^{-1}(11)$

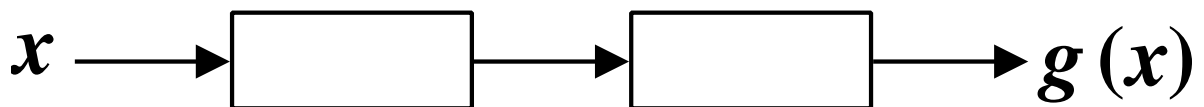
(iii)  $f(9)$

(iv)  $f^{-1}(29)$

**Question 8.**

- (a) Fill in the flow diagram for the function

$$g(x) = \frac{x - 5}{4}$$



- (b) Write down  $g^{-1}(x)$

- (c) Determine the value of,

(i)  $g(13)$

(ii)  $g^{-1}(11)$

(iii)  $g(37)$

(iv)  $g^{-1}(7)$

**Question 9.**

Consider the function,

$$f(x) = 8x + 3$$

Find an expression for the inverse function  $f^{-1}(x)$

**HINT :** Draw a flow diagram.

**Question 10.**

Consider the function,

$$g(x) = \frac{x}{7} + 3$$

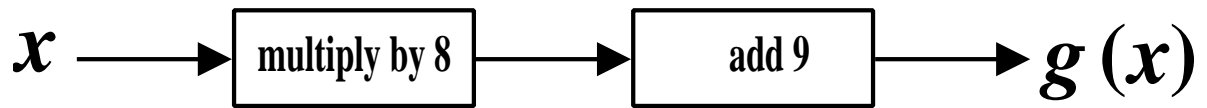
Find an expression for the inverse function  $g^{-1}(x)$

**HINT :** Draw a flow diagram.



### 6.3 Homework

#### Question 1.



(a) Write down (i)  $g(x)$

(ii)  $g^{-1}(x)$

(b) Determine the value of,

(i)  $g(5)$

(ii)  $g^{-1}(33)$

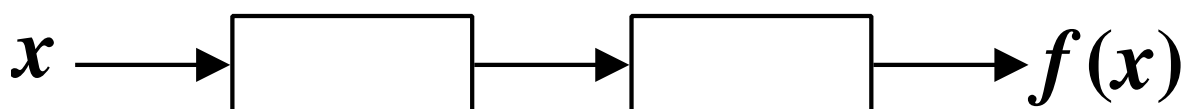
(iii)  $g(0.5)$

(iv)  $g^{-1}(1)$

#### Question 2.

(a) Fill in the flow diagram for the function

$$f(x) = 3x - 7$$



(b) Write down  $f^{-1}(x)$

(c) Determine the value of,

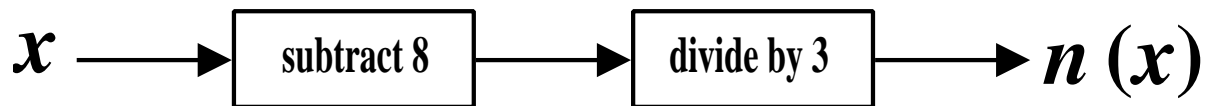
(i)  $f(12)$

(ii)  $f^{-1}(11)$

(iii)  $f(1)$

(iv)  $f^{-1}(29)$

**Question 3.**



(a) Write down (i)  $n(x)$

(ii)  $n^{-1}(x)$

(b) Determine the value of,

(i)  $n(23)$

(ii)  $n^{-1}(9)$

(iii)  $n(41)$

(iv)  $n^{-1}(-2)$

**Question 4.**

Consider the function,

$$k(x) = 9x + 7$$

Find an expression for the inverse function  $k^{-1}(x)$

**HINT :** Draw a flow diagram.

## 6.4 Answers

### 6.4.1 Solutions (Introductory Example)

$$f(x) = 5x + 4 \qquad f^{-1}(x) = \frac{x - 4}{5}$$

- (i) 44                      (ii) 5  
(iii) 59                    (iv) 8

8 in gives 44 out so in the inverse 44 in gets back to the 8.

### 6.4.2 Solutions (6.2 Exercise)

#### Answer 1.

- (a) (i)  $6x + 5$                       (ii)  $\frac{x - 5}{6}$   
(b) (i) 29                              (ii) 6  
      (iii) 59                            (iv) 11

#### Answer 2.

- (a) (i)  $6(x + 5)$                       (ii)  $\frac{x}{6} - 5$   
       $= 6x + 30$   
(b) (i) 54                              (ii) 3  
      (iii) 120                           (iv) 7

#### Answer 3.

- (a) (i)  $7(x - 8)$                       (ii)  $\frac{x}{7} + 8$   
       $= 7x - 56$   
(b) (i) 35                              (ii) 19  
      (iii) 140                           (iv) 78

#### Answer 4.

- (a) (i)  $8x - 5$                       (ii)  $\frac{x + 5}{8}$   
(b) (i) 43                              (ii) 3  
      (iii) 7                              (iv) 11

#### Answer 5.

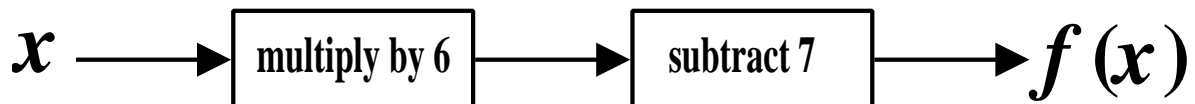
- (a) (i)  $\frac{x}{3} + 11$                       (ii)  $3(x - 11)$   
       $= 3x - 33$   
(b) (i) 33                              (ii) 12  
      (iii) 18                            (iv) 24

**Answer 6.**

- |     |       |                 |      |        |
|-----|-------|-----------------|------|--------|
| (a) | (i)   | $\frac{x-7}{5}$ | (ii) | $5x+7$ |
| (b) | (i)   | 3               | (ii) | 12     |
|     | (iii) | 6               | (iv) | 47     |

**Answer 7.**

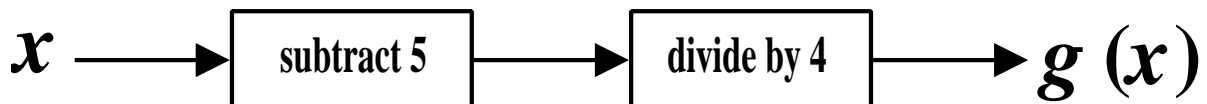
(a)



- |     |                 |    |      |   |
|-----|-----------------|----|------|---|
| (b) | $\frac{x+7}{6}$ |    |      |   |
| (c) | (i)             | 53 | (ii) | 3 |
|     | (iii)           | 47 | (iv) | 6 |

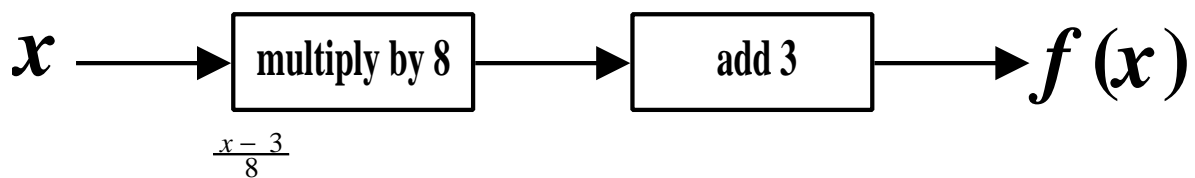
**Answer 8.**

(a)

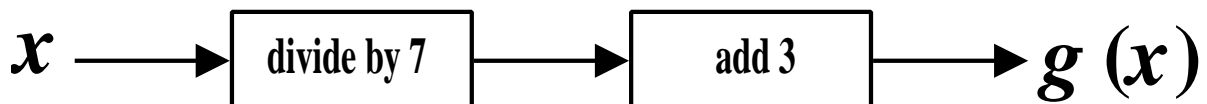


- |     |        |   |      |    |
|-----|--------|---|------|----|
| (b) | $4x+5$ |   |      |    |
| (c) | (i)    | 2 | (ii) | 49 |
|     | (iii)  | 8 | (iv) | 33 |

**Answer 9.**



**Answer 10.**



$7(x-3)$  or  $7x-21$

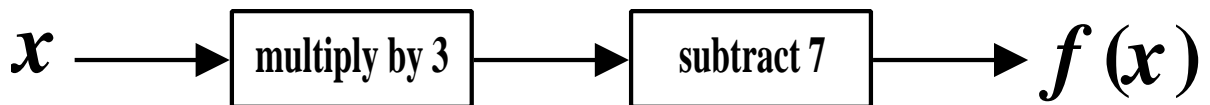
### 6.4.3 Solutions (6.3 Homework)

#### Answer 1.

- |     |       |          |      |                 |
|-----|-------|----------|------|-----------------|
| (a) | (i)   | $8x + 9$ | (ii) | $\frac{x-9}{8}$ |
| (b) | (i)   | 49       | (ii) | 3               |
|     | (iii) | 13       | (iv) | -1              |

#### Answer 2.

(a)

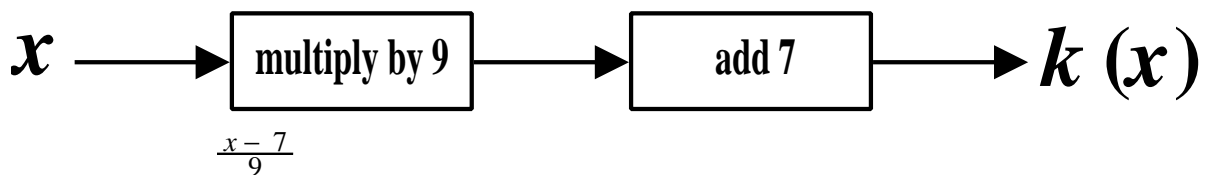


- |     |                 |    |      |    |
|-----|-----------------|----|------|----|
| (b) | $\frac{x+7}{3}$ |    |      |    |
| (c) | (i)             | 29 | (ii) | 6  |
|     | (iii)           | -4 | (iv) | 12 |

#### Answer 3.

- |     |       |                 |      |          |
|-----|-------|-----------------|------|----------|
| (a) | (i)   | $\frac{x-8}{3}$ | (ii) | $3x + 8$ |
| (b) | (i)   | 5               | (ii) | 35       |
|     | (iii) | 11              | (iv) | 2        |

#### Answer 4.



## 7.1 More involved Inverse Functions from Flow Diagrams

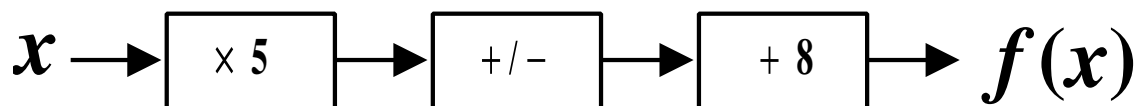
Previously we've considered flow diagrams involving two action boxes. More than two action boxes may be involved.

Previously the action boxes contained the arithmetical operations  $+$ ,  $-$ ,  $\times$  and  $\div$ . Other arithmetical operations may be involved.

**Example**

The following flow diagram contains the action boxes,

- ◇ Multiply by 5
- ◇ Change sign
- ◇ Add 8



Write down the function described by the flow diagram, and also the inverse function.

$$f(x) =$$



$$f^{-1}(x) =$$



Having extracted the algebra we need from the flow diagram we can now answer a couple of easy questions.

Determine the value of,

(i)  $f(1)$

(ii)  $f^{-1}(3)$

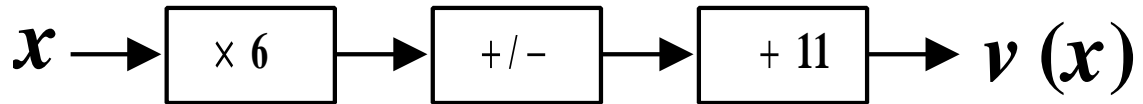


**DID YOU NOTICE ?** : The *reverse* of *Change sign* was *Change sign*.

## 7.2 Exercise

### Question 1.

The flow diagram action boxes are *Multiply by 6*, *Change sign* and *Add 11*.



(a) Write down (i)  $v(x)$

(ii)  $v^{-1}(x)$

(b) Determine the value of,

(i)  $v(1)$

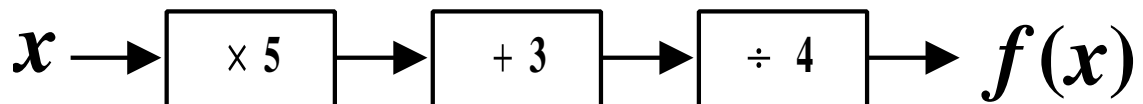
(ii)  $v^{-1}(23)$

(iii)  $v(-2)$

(iv)  $v^{-1}(-1)$

### Question 2.

The flow diagram action boxes are *Multiply by 5*, *Add 3* and *Divide by 4*.



(a) Write down (i)  $f(x)$

(ii)  $f^{-1}(x)$

(b) Determine the value of,

(i)  $f(5)$

(ii)  $f^{-1}(12)$

(iii)  $f(-3)$

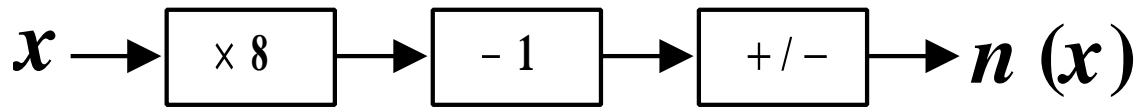
(iv)  $f^{-1}(-8)$





**Question 5.**

The flow diagram action boxes are *Multiply by 8*, *Subtract 1* and *Change sign*.



(a) Write down (i)  $n(x)$

(ii)  $n^{-1}(x)$

(b) Determine the value of,

(i)  $n(3)$

(ii)  $n^{-1}(1)$

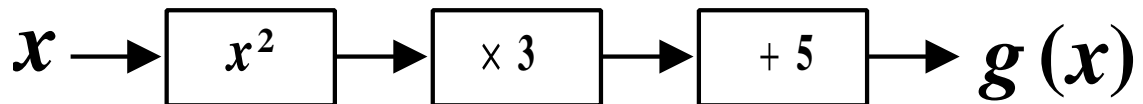
(iii)  $n(-1)$

(iv)  $n^{-1}(-7)$

**Question 6.**

The flow diagram action boxes are *Square*, *Multiply by 3* and *Add 5*.

The domain is restricted so that only positive integers are to be considered.



(a) Write down (i)  $g(x)$

(ii)  $g^{-1}(x)$

(b) Determine the value of,

(i)  $g(10)$

(ii)  $g^{-1}(8)$

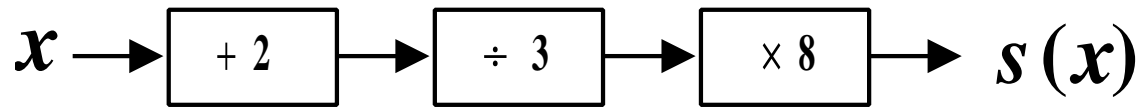
(iii)  $g(5)$

(iv)  $g^{-1}(53)$

(c) Why was the domain in this question restricted ?

**Question 7.**

The flow diagram action boxes are *Add 2*, *Divide by 3* and *Multiply by 8*.



(a) Write down (i)  $s(x)$

(ii)  $s^{-1}(x)$

(b) Determine the value of,

(i)  $s(1)$

(ii)  $s^{-1}(16)$

(iii)  $s(7)$

(iv)  $s^{-1}(72)$

### 7.3 Answers

#### 7.3.1 Solutions ( Introductory example )

$$f(x) = -5x + 8$$

$$f^{-1}(x) = \frac{-x + 8}{5}$$

(i)  $f(1) = 3$

(ii)  $f^{-1}(3) = 1$

#### 7.3.2 Solutions ( 7.2 Exercise )

##### Answer 1.

(a) (i)

$$v(x) = -6x + 11$$

(ii)

$$v^{-1}(x) = \frac{-x + 11}{6}$$

(b) (i) 5  
(iii) 23

(ii) -2  
(iv) 2

##### Answer 2.

(a) (i)

$$f(x) = \frac{5x + 3}{4}$$

(ii)

$$f^{-1}(x) = \frac{4x - 3}{5}$$

(b) (i) 7  
(iii) -3

(ii) 9  
(iv) -7

##### Answer 3.

(a) (i)

$$h(x) = 4\sqrt{x} - 7$$

(ii)

$$h^{-1}(x) = \left(\frac{x + 7}{4}\right)^2$$

(b) (i) 13  
(iii) 29

(ii) 9  
(iv) 256

**Answer 4.****(a) (i)**

$$k(x) = -7x + 21$$

**(ii)**

$$k^{-1}(x) = -\frac{x}{7} + 3$$

**(b) (i) 7****(iii) -14****(ii) 2****(iv) 6****Answer 5.****(a) (i)**

$$n(x) = -8x + 1$$

**(ii)**

$$n^{-1}(x) = \frac{-x + 1}{8}$$

**(b) (i) -23****(iii) 9****(ii) 0****(iv) 1****Answer 6.****(a) (i)**

$$n(x) = 3x^2 + 5$$

**(ii)**

$$n^{-1}(x) = \sqrt{\frac{x - 5}{3}}$$

**(b) (i) 305****(iii) 80****(ii) 1****(iv) 4**

**(c)** Otherwise not know if the reverse of  $x^2$  is  $+\sqrt{x}$  or  $-\sqrt{x}$   
 Furthermore, for the inverse,  $x \geq 5$  to avoid taking a negative square root.

**Answer 7.****(a) (i)**

$$s(x) = \frac{8(x + 2)}{3}$$

**(ii)**

$$s^{-1}(x) = \frac{3x}{8} - 2$$

**(b) (i) 8****(iii) 24****(ii) 4****(iv) 25**

**8.1 Inverse Functions without Flow Diagrams**

There is a more mathematical method that can be used to find the inverse of a function.

Using it means that there solutions are less cluttered.

There are also functions which cannot be represented by flow diagrams.

The mathematical method allows their inverses to be found also.

**8.2 Example**

$$f(x) = \frac{5}{2x} + 6$$

(i) Determine  $f(5)$

(ii) Find  $f^{-1}(x)$

(iii) Use your part (i) answer to check your part (ii) answer.

### 8.3 Exercise

In each question find the inverse function;

(i)  $f(x) = 7 - 3x$

(ii)  $g(x) = 8x + 3$

(iii)  $h(x) = \frac{1}{2}x + 5$

(iv)  $k(x) = \frac{x}{5} - 4$

$$(v) \quad m(x) = 2(3 - 5x)$$

$$(vi) \quad n(x) = \frac{x - 8}{3}$$

$$(vii) \quad p(x) = \frac{7x}{4}$$

$$(viii) \quad q(x) = \frac{3x}{5} + 4$$

(ix)  $r(x) = \frac{x}{11}$

(x)  $s(x) = \frac{1}{4x}, x \neq 0$

(xi)  $t(x) = \frac{3}{2x}, x \neq 0$

(xii)  $u(x) = \frac{1}{x} + 4, x \neq 0$



(xiii)  $v(x) = 9 - \frac{1}{x}, x \neq 0$

(xiv)  $w(x) = 5 - \frac{3}{x}, x \neq 0$

(xv)  $z(x) = \frac{2}{x} + 5, x \neq 0$

(xvi)  $a(x) = \frac{x+4}{x}, x \neq 0$

(xvii)  $b(x) = \frac{7-x}{x}, x \neq 0$

(xviii)  $c(x) = \frac{5-3x}{x}, x \neq 0$

(**xix**)  $d(x) = \frac{x+3}{x+2}, x \neq -2$

(**xx**)  $e(x) = \frac{x+1}{x-2}, x \neq 2$

$$\text{(xxi)} \quad f(x) = \frac{2}{4x} + \frac{1}{3x}, \quad x \neq 0$$

$$\text{(xxii)} \quad g(x) = \frac{3}{5x} + \frac{1}{4x} + 2, \quad x \neq 0$$

$$( \text{xxiii} ) \quad h(x) = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}, \quad x \neq -4, -2$$

HINT: factorise

$$( \text{xxiv} ) \quad k(x) = \frac{x^2 - 4}{x^2 - 4x + 4}, \quad x \neq 2$$

## Chapter 9

## Functions : GCSE

### 9.1 Revision

#### Question 1.

The function  $f$  is given by;

$$f(x) = x^2 - 1$$

Calculate;

(i)  $f(3)$

(ii)  $f(8)$

(iii)  $f(1)$

(iv)  $f(0)$

(v)  $f(5)$

(vi)  $f(-5)$

(vii)  $f(100)$

(viii)  $f\left(\frac{1}{2}\right)$

(ix)  $f\left(\frac{3}{2}\right)$

(x)  $f(\sqrt{2})$

#### Question 2.

Sometimes the domain of a function is restricted.

Consider the function,

$$g(x) = \frac{10}{x+1} \quad x \neq -1$$

(i) What number is not allowed into this function ?

(ii) Why is this function's domain restricted in this way ?

**Question 3.**

This question involves the functions;

$$f(x) = 3x - 2 \quad \& \quad g(x) = \frac{10}{x + 1} \quad x \neq -1$$

Determine the value of,

(i)  $f(13)$

(ii)  $g(0)$

(iii)  $ff(3)$

(iv)  $fg(4)$

(v)  $gf(1)$

**Question 4.**

If  $v(x) = 3x^2 - 1$ , find expressions that do not involve brackets for;

(i)  $v(7)$

(ii)  $v(10x)$

(iii)  $v(x + 4)$

**HINT : FOIL**

**Question 5.**

If  $m(x) = 6x + 7$ , find  $x$  such that  $m(x) = 25$ .

**Question 6.**

If  $s(x) = \frac{3x + 2}{4}$ , find  $x$  such that  $s(x) = 11$ .

**Question 7.**

Let  $p$  and  $q$  be the functions;

$$p(x) = 4x + 7$$

$$q(x) = 5x + 6$$

Evaluate each of the following;

(i)  $p q(3)$

(ii)  $p q(-1)$

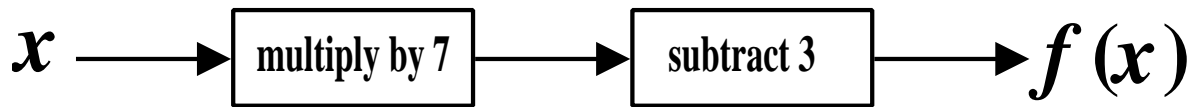
(iii)  $p q(4z)$

(iv)  $p q(3z + 1)$



**Question 8.**

Consider the following flow diagram;



(a) Write down (i)  $f(x)$

(ii)  $f^{-1}(x)$

(b) Determine the value of,

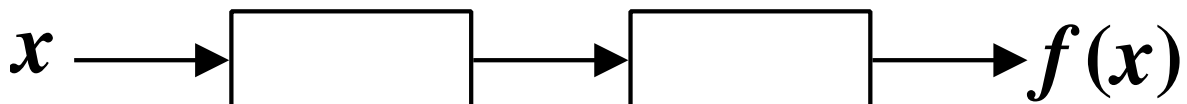
(i)  $f(5)$

(ii)  $f^{-1}(32)$

**Question 9.**

(a) Fill in the flow diagram for the function

$$f(x) = 5(x - 9)$$



(b) Write down  $f^{-1}(x)$

(c) Determine the value of,

(i)  $f(12)$

(ii)  $f^{-1}(25)$

**Question 10.**

Consider the function,

$$k(x) = \frac{x}{5} + 7$$

Find an expression for the inverse function  $k^{-1}(x)$

**HINT :** Draw a flow diagram.

*The GCSE examination often includes an awkward functions question.  
Here is an example of an A\* grade question.*

**Question 11.**

For the function  $f(x) = 3x + 2$ , determine  $x$  such that  $f(x) = f^{-1}(x)$ .

**Question 12.**

Consider the function,

$$f(x) = \frac{8}{3x} + 7$$

Find an expression for the inverse function  $f^{-1}(x)$

**Question 13.**

Consider the function,

$$f(x) = \frac{x + 5}{x + 3}$$

Find an expression for the inverse function  $f^{-1}(x)$

## 9.2 Solutions to the Revision

### Answer 1.

- (i) 8      (ii) 63      (iii) 0      (iv) -1      (v) 24  
(vi) 24      (vii) 9999      (viii)  $-\frac{3}{4}$       (ix)  $\frac{5}{4}$       (x) 1

### Answer 2.

- (i) -1      (ii) To avoid a division by zero (which is not defined).

### Answer 3.

- (i) 37      (ii) 10      (iii) 19      (iv) 4      (v) 5

### Answer 4.

- (i) 146      (ii)  $300x^2 - 1$       (iii)  $3x^2 + 24x + 47$

### Answer 5.

3

### Answer 6.

14

### Answer 7.

- (i) 91      (ii) 11      (iii)  $80z + 31$       (iv)  $60z + 51$

### Answer 8.

- (a) (i)

$$f(x) = 7x - 3$$

- (b) (i) 32

- (ii)

$$f^{-1}(x) = \frac{x + 3}{7}$$

- (ii) 5

### Answer 9.

- (a) Diagram contains *Subtract 9*, then *Multiply by 5*.

(b)  $\frac{x}{5} + 9$

- (c) (i) 15

- (ii) 14

### Answer 10.

$$5x - 35$$

### Answer 11.

-1

### Answer 12.

$$f^{-1}(x) = \frac{8}{3(x - 7)}$$

### Answer 13.

$$f^{-1}(x) = \frac{5 - 3x}{x - 1}$$

## Chapter 10

## Functions : GCSE

### 10.1 Homework

#### Question 1.

The function  $f$  is given by;

$$f(x) = x^2 + 3$$

Calculate;

(i)  $f(4)$

(ii)  $f(9)$

(iii)  $f(1)$

(iv)  $f(0)$

(v)  $f(6)$

(vi)  $f(-6)$

(vii)  $f(100)$

(viii)  $f\left(\frac{1}{2}\right)$

(ix)  $f\left(\frac{3}{2}\right)$

(x)  $f(\sqrt{5})$

#### Question 2.

Sometimes the domain of a function is restricted.

Consider the function,

$$g(x) = \frac{12}{x+2} \quad x \neq -2$$

(i) What number is not allowed into this function ?

(ii) Why is this function's domain restricted in this way ?

**Question 3.**

This question involves the functions;

$$f(x) = 5x - 3 \quad \& \quad g(x) = \frac{12}{x + 2} \quad x \neq -2$$

Determine the value of,

(i)  $f(8)$

(ii)  $g(1)$

(iii)  $ff(3)$

(iv)  $fg(4)$

(v)  $gf(1)$

**Question 4.**

If  $H(x) = 4x^2 - 1$ , find expressions that do not involve brackets for;

(i)  $H(7)$

(ii)  $H(10x)$

(iii)  $H(x + 3)$

**HINT : FOIL**

**Question 5.**

If  $w(x) = 8x + 7$ , find  $x$  such that  $w(x) = 31$ .

**Question 6.**

If  $n(x) = \frac{2x + 5}{3}$ , find  $x$  such that  $n(x) = 13$ .

**Question 7.**

Let  $u$  and  $v$  be the functions;

$$u(x) = 7x + 4$$

$$v(x) = 6x + 5$$

Evaluate each of the following;

(i)  $uv(1)$

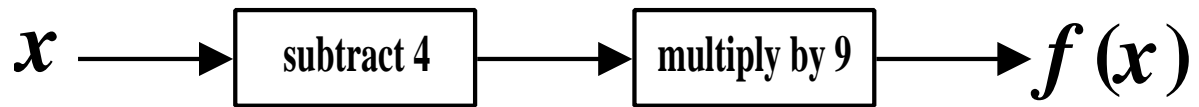
(ii)  $uv(-1)$

(iii)  $uv(3z)$

(iv)  $uv(4z + 1)$

**Question 8.**

Consider the following flow diagram;



(a) Write down (i)  $f(x)$

(ii)  $f^{-1}(x)$

(b) Determine the value of,

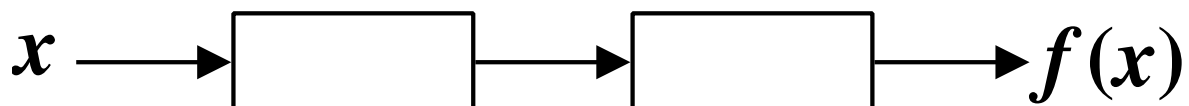
(i)  $f(8)$

(ii)  $f^{-1}(45)$

**Question 9.**

(a) Fill in the flow diagram for the function

$$L(x) = 7(x - 8)$$



(b) Write down  $L^{-1}(x)$

(c) Determine the value of,

(i)  $L(17)$

(ii)  $L^{-1}(35)$



**Question 10.**

Consider the function,

$$k(x) = \frac{x}{7} - 3$$

Find an expression for the inverse function  $k^{-1}(x)$

**HINT :** Draw a flow diagram.

*The GCSE examination often includes an awkward functions question.  
Here is an example of an A\* grade question.*

**Question 11.**

For the function  $f(x) = 4x - 1$ , determine  $x$  such that  $f(x) = f^{-1}(x)$ .

**Question 12.**

Consider the function,

$$f(x) = \frac{13}{5x} + 4$$

Find an expression for the inverse function  $f^{-1}(x)$

**Question 13.**

Consider the function,

$$f(x) = \frac{x + 11}{x - 2}$$

Find an expression for the inverse function  $f^{-1}(x)$

## 10.2 Solutions to the Homework.

### Answer 1.

- (i) 19      (ii) 84      (iii) 4      (iv) 3      (v) 39  
(vi) 39      (vii) 10003      (viii) 3.25      (ix) 5.25      (x) 8

### Answer 2.

- (i) -2      (ii) To avoid a division by zero (which is not defined).

### Answer 3.

- (i) 37      (ii) 4      (iii) 57      (iv) 7      (v) 3

### Answer 4.

- (i) 195      (ii)  $400x^2 - 1$       (iii)  $4x^2 + 24x + 35$

### Answer 5.

3

### Answer 6.

17

### Answer 7.

- (i) 81      (ii) -3      (iii)  $126z + 109$       (iv)  $168z + 81$

### Answer 8.

- (a) (i)

$$f(x) = 9x - 36$$

- (b) (i) 36

- (ii)

$$f^{-1}(x) = \frac{x}{9} + 4$$

- (ii) 9

### Answer 9.

- (a) Diagram contains *Subtract 8*, then *Multiply by 7*.

(b)  $\frac{x}{7} + 8$

- (c) (i) 63

- (ii) 13

### Answer 10.

$$7x + 21$$

### Answer 11.

$$\frac{1}{3}$$

### Answer 12.

$$f^{-1}(x) = \frac{13}{5(x-4)}$$

### Answer 13.

$$f^{-1}(x) = \frac{11 + 2x}{x - 1}$$

## Chapter 11

## Functions : GCSE

### 11.1 TEST

#### Question 1.

The function  $f$  is given by;

$$f(x) = x^2 - x$$

Calculate;

(i)  $f(7)$

(ii)  $f(3)$

(iii)  $f(10)$

(iv)  $f(0)$

(v)  $f(1)$

(vi)  $f(4)$

(vii)  $f(-4)$

(viii)  $f(100)$

(ix)  $f\left(\frac{1}{2}\right)$

(x)  $f\left(-\frac{1}{2}\right)$

#### Question 2.

Sometimes the domain of a function is restricted.

Consider the function,

$$g(x) = \frac{12}{x-3} \quad x \neq 3$$

(i) What number is not allowed into this function ?

(ii) Why is this function's domain restricted in this way ?

**Question 3.**

This question involves the functions;

$$f(x) = 4x - 1 \quad \& \quad g(x) = \frac{12}{x - 3} \quad x \neq 3$$

Determine the value of,

(i)  $f(8)$

(ii)  $g(5)$

(iii)  $ff(5)$

(iv)  $fg(9)$

(v)  $gf(2)$

**Question 4.**

If  $H(x) = 3x^2 + 1$ , find expressions that do not involve brackets for;

(i)  $H(6)$

(ii)  $H(10x)$

(iii)  $H(x + 4)$

**HINT : FOIL**

**Question 5.**

If  $w(x) = 7x + 5$ , find  $x$  such that  $w(x) = 33$ .

**Question 6.**

If  $n(x) = \frac{3x + 2}{5}$ , find  $x$  such that  $n(x) = 7$ .

**Question 7.**

Let  $u$  and  $v$  be the functions;

$$u(x) = 4x + 7$$

$$v(x) = 5x + 6$$

Evaluate each of the following;

(i)  $uv(1)$

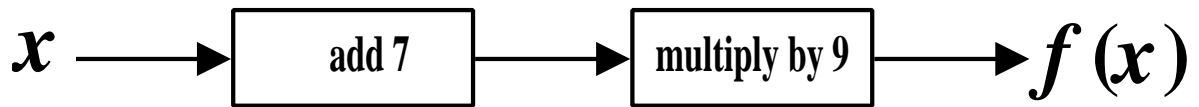
(ii)  $uv(-1)$

(iii)  $uv(5z)$

(iv)  $uv(3z + 2)$

**Question 8.**

Consider the following flow diagram;



(a) Write down (i)  $f(x)$

(ii)  $f^{-1}(x)$

(b) Determine the value of,

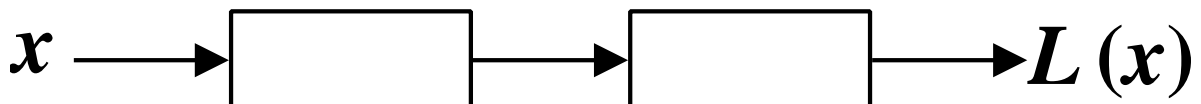
(i)  $f(-3)$

(ii)  $f^{-1}(45)$

**Question 9.**

(a) Fill in the flow diagram for the function

$$L(x) = 11(x - 4)$$



(b) Write down  $L^{-1}(x)$

(c) Determine the value of,

(i)  $L(24)$

(ii)  $L^{-1}(44)$

**Question 10.**

Consider the function,

$$k(x) = \frac{x}{13} - 7$$

Find an expression for the inverse function  $k^{-1}(x)$

**HINT :** Draw a flow diagram.

*The GCSE examination often includes an awkward functions question.  
Here is an example of an A\* grade question.*

**Question 11.**

For the function  $f(x) = 5x + 1$ , determine  $x$  such that  $f(x) = f^{-1}(x)$ .



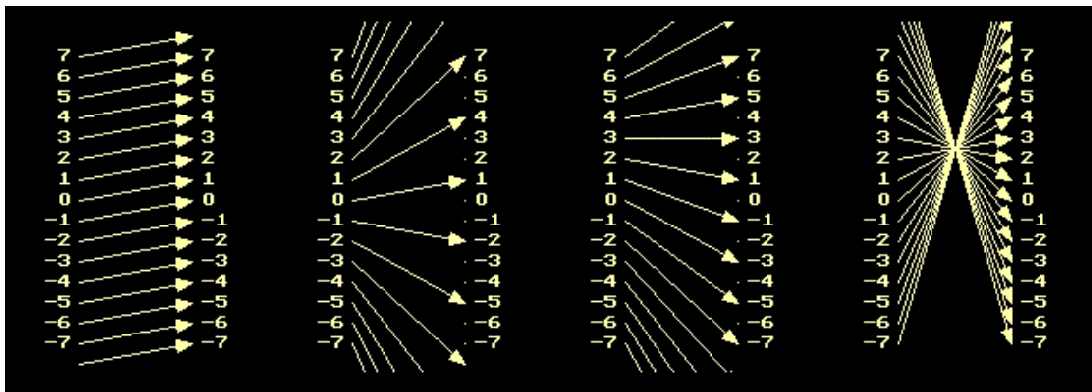
**Question 12.**

If  $p(x) = \frac{5x-7}{3-x}$ , find  $x$  such that  $p(x) = 11$

**Question 13.**

Four arrow diagrams are shown below.

- (i) Which diagram is a mapping of the function,  $w(x) = 5 - x$  ?
- (ii) For each of the other three diagrams, suggest the functions being mapped.



**Question 14.**

If  $f(x) = 3x + 5$  and  $g(x) = x^2 + 5$ ,

Find; (i)  $fg(4)$

(ii)  $gff(-1)$

(iii)  $gf(x)$  : Write your answer without using brackets.

**Question 15.**

If  $v(x) = 10 - x$  and  $w(x) = \sqrt{x}$ ,

(a) Find; (i)  $vw(36)$

(ii)  $wv(-1)$

(iii)  $wv(x)$

(b) If  $vw(x) = 0$ , what must be the value of  $x$  ?

**Question 16.**

Consider the function,

$$f(x) = 8 - \frac{12}{7x}$$

Find an expression for the inverse function  $f^{-1}(x)$

**Question 17.**

Consider the function,

$$f(x) = \frac{13 - x}{5 + 2x}$$

Find an expression for the inverse function  $f^{-1}(x)$