A-Level Applied Mathematics

Mechanics 1

Topic N° 1

K • I • N • E • M • A • T • I • C • S

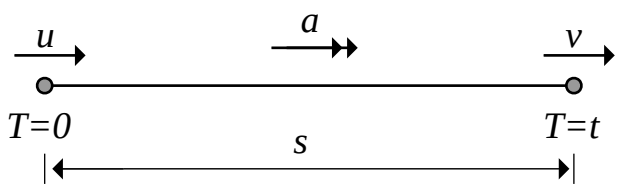
K I N E M A T I C S

Chapter 1

Kinematics : Mechanics 1
Additional Mathematics

1.1 Derivation of the SUVAT equations

Consider a straight line interval over which a particle accelerates uniformly;



s = *displacement*

u = *initial velocity*

v = *final velocity*

a = *acceleration* (*constant*)

t = *time*

We know that acceleration is is the rate of change of velocity.
Furthermore, if the acceleration is constant,

$$a = \frac{change\ in\ velocity}{change\ in\ time}$$

$$a = \frac{v - u}{t - 0}$$

$$at = v - u$$

$$\therefore\ v = u + at \qquad \textit{Result } N^\circ 1$$

When the acceleration is zero, we know that

$$Distance = Speed \times Time$$

but when there is constant acceleration this becomes

$$Displacement = Average\ Speed \times Time$$

$$s = \left(\frac{v + u}{2}\right) t \qquad \textit{Result } N^\circ 2$$

Substituting *Result N°* 1 into *Result N°* 2 gives *Result N°* 3 as follows;

$$s = \left(\frac{v + u}{2} \right) t$$

$$s = \left(\frac{(u + at) + u}{2} \right) t$$

$$s = \left(\frac{2u + at}{2} \right) t$$

$$s = \left(\frac{2u}{2} + \frac{at}{2} \right) t$$

$$s = \left(u + \frac{1}{2} at \right) t$$

$$s = ut + \frac{1}{2} a t^2 \qquad \textit{Result N°} 3$$

A fourth result is obtained by first noting that *Result N°* 1

$$v = u + at$$

can be written as

$$t = \frac{v - u}{a}$$

and substituting this into *Result N°* 2

$$s = \left(\frac{v + u}{2} \right) t$$

$$s = \left(\frac{v + u}{2} \right) \left(\frac{v - u}{a} \right)$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \qquad \textit{Result N°} 4$$

A fifth result is obtained by first noting that *Result N°* 1

$$v = u + at$$

can be written as

$$u = v - at$$

and substituting this into *Result N°* 2

$$s = \left(\frac{v + u}{2} \right) t$$

$$s = \left(\frac{v + v - at}{2} \right) t$$

$$s = \left(\frac{2v - at}{2} \right) t$$

$$s = \left(\frac{2v}{2} - \frac{at}{2} \right) t$$

$$s = \left(v - \frac{1}{2} at \right) t$$

$$s = vt - \frac{1}{2} a t^2 \qquad \textit{Result N°} 5$$

1.2 The five SUVAT equations

$v = u + at$	s
$s = vt - \frac{1}{2} a t^2$	u
$s = ut + \frac{1}{2} a t^2$	v
$s = \left(\frac{v + u}{2} \right) t$	a
$v^2 = u^2 + 2as$	t

1.3 Example

Additional Mathematics Examination Question from June 2010, Q6

An aeroplane touches down at a point *A* on a runway, travelling at 90 m s^{-1} It then decelerates uniformly until it reaches a speed of 6 m s^{-1} at a point *B* on the runway, 2016 m from *A*.

(i) Find the deceleration.

[3 marks]

(ii) Find the time taken to travel from *A* to *B*.

[2 marks]

1.4 Exercise

Question 1.

Additional Mathematics Examination Question from June 2007, Q5

A car is travelling along a motorway at 30 m s^{-1}. At the moment that it passes a point *A* the brakes are applied so that the car decelerates with constant deceleration. When it reaches a point *B*, where *AB* = 300 m, the speed of the car is 10 m s^{-1}. Calculate

(i) the constant deceleration

[3 marks]

(ii) the time taken to travel from *A* to *B*.

[2 marks]

Question 2.

Additional Mathematics Examination Question from June 2009, Q5

Parcels slide down a ramp.

Due to resistance the deceleration is 0.25 m s^{-2}

(i) One parcel is given an initial velocity of 2 m s^{-1}
Find the distance travelled before the parcel comes to rest.

[3 marks]

(ii) A second parcel is given an initial velocity of 3 m s^{-1} and takes 4 seconds to reach the bottom of the ramp.
Find the length of the ramp.

Be careful : The parcel will be moving as it flies off the end of the ramp !

[3 marks]

Question 3.
Without looking them up, try to write out all five *suvat* equations from memory.

Question 4.
Additional Mathematics Examination Question from June 2003, Q10
A car, which is initially travelling at 20 m s^{-1}, accelerates uniformly at 1.2 m s^{-2}
Find
(i) the speed after 5 seconds

[2 marks]

(ii) the distance travelled in this time.

[2 marks]

Question 5.

A particle moving in a straight line experiences a constant retardation of 6 $m.s^{-2}$
It passes a point, *A*, with initial velocity 25 $m.s^{-1}$
As it passes *A* a stopwatch is started.

(i) What will the stopwatch read as the particle first has a displacement of 28 metres from *A* ?

(ii) What will the stopwatch read when the particle has a displacement of 28 metres from *A* for the second time ?

(iii) There is a third moment when the particle is at a ***distance*** of of 28 metres from *A.*
Determine the stopwatch reading for this third moment.

Chapter 2

Kinematics : Mechanics 1

2.1 Horizontal Kinematics Questions

In Chapter 3 we will look at questions in which particles are thrown vertically upwards. In such questions we have to consider the importance of gravity.

In this chapter, however, we shall focus on horizontal motion in which gravity does not come into the method of solution.

2.2 Exercise

Question 1.

M1 examination question, May 2002, Q1

A car moves with constant acceleration along a straight horizontal road

The car passes the point A with speed 5 ms^{-1} and 4 s later it passes the point B

AB = 50 m.

(a) Find the acceleration of the car

[3 marks]

When the car passes the point C, it has speed 30 ms^{-1}

(b) Find the distance AC

[3 marks]

Question 2.

M1 examination question, June 2005, Q1

In taking off, an aircraft moves on a straight runway *AB* of length 1.2 km
The aircraft moves from *A* with initial speed 2 ms^{-1}
It moves with constant acceleration and 20 s later it leaves the runway at *C* with a speed 74 ms^{-1}
Find

(a) the acceleration of the aircraft

[2 marks]

(b) Find the distance *BC*

[4 marks]

Question 3.

M1 examination question, Mock Paper 2000, Q1

An aircraft moves along a straight runway with constant acceleration.
It passes a point *A* on the runway with speed 16 ms^{-1}
It then passes the point *B* on the runway with speed 34 ms^{-1}
The distance from *A* to *B* is 150 m

(a) Find the acceleration of the aircraft

[3 marks]

(b) Find the time taken by the aircraft in moving from *A* to *B*

[2 marks]

(c) Find, to 3 significant figures, the speed of the aircraft when it passes the point mid-way between *A* and *B*

[2 marks]

Question 4.

M1 examination question, Specimen Paper 2000, Q2

A car starts from rest at a point *O* and moves in a straight line.

The car moves with constant acceleration 4 ms^{-2} until it passes the point *A* when it is moving with speed 10 ms^{-1}

It then moves with constant acceleration 3 ms^{-2} for 6 s until it reaches the point *B*

(a) Find the speed of the car at *B*

[2 marks]

(b) Find the distance *OB*

[5 marks]

Question 5.

M1 examination question, January 2002, Q3

A racing car moves with constant acceleration along a straight horizontal road.
It passes the point *O* with speed 12 ms^{-1}
It passes the point *A* 4 s later with speed 60 ms^{-1}

(a) Show that the acceleration of the car is 12 ms^{-2}

[2 marks]

(b) Find the distance *OA*

[3 marks]

The point *B* is the mid-point of *OA*

(c) Find, to 3 significant figures, the speed of the car when it passes *B*

[3 marks]

Question 6.

M1 examination question, June 2006, Q3

A train moves along a straight track with constant accelleration.

Three telegraph poles are set at equal intervals beside the track at points *A*, *B* and *C*

AB = 50 m and *BC* = 50 m

The front of the train passes *A* with speed 22.5 ms^{-1} and 2 s later it passes *B*

Find

(a) the acceleration of the train

[3 marks]

(b) the speed of the front of the train when it passes *C*

[3 marks]

(c) the time that elapses from the instant the front of the train passes *B* to the instant it passes *C*

[4 marks]

Question 7.

M1 examination question, May 2009, Q1

Three posts *P*, *Q* and *R*, are fixed in that order at the side of a straight horizontal road
The distance from *P* to *Q* is 45 m and the distance from *Q* to *R* is 120m
A car is moving along the road with constant acceleration a ms^{-2}
The speed of the car, as it passes *P* is u ms^{-1}
The car passes *Q* two seconds after passing *P* and *R* four seconds after passing *Q*

Find

(a) the value of u

(b) the value of a

[7 marks]

Chapter 3

Kinematics : Mechanics 1

3.1 Projectiles

$$v = u + at \qquad s$$

$$s = vt - \frac{1}{2} a t^2 \qquad u$$

$$s = ut + \frac{1}{2} a t^2 \qquad v$$

$$s = \left(\frac{v + u}{2} \right) t \qquad a$$

$$v^2 = u^2 + 2as \qquad t$$

The five suvat equations **ASSUME CONSTANT ACCELERATION**

Fortunately, there are many situations in the physical world where the acceleration is indeed constant. One useful 'real world' application of the suvat equations is in analysing a projectile.

Discuss : What is a projectile ?

3.2 Worked example

It is Chloe's birthday.
To help her celebrate her eleven best friends collectively chuck her into the air.
Chloe weighs 43 kg and she is launched vertically upwards at 4 ms^{-1}

(i) How high does she go ?

(ii) For how long is she aloft ?

(iii) Did Chloe's weight play any part in this question's calculations ?

3.3 An Examination Question

M1 examination question, May 2008, Q2

At time $t = 0$, a particle is projected vertically upwards with speed u ms^{-1} from a point 10 m above the ground. At time T seconds, the particle hits the ground with speed 17.5 ms^{-1}
Find

(a) the value of u

[3 marks]

(b) the value of T

[4 marks]

3.4 Exercise

Question 1.

M1 examination question, May 2003, Q2

A competitor makes a dive from a high springboard into a diving pool.
She leaves the springboard vertically with a speed of 4 ms^{-1} upwards.
When she leaves the springboard, she is 5 m above the surface of the pool.
The diver is modelled as a particle moving vertically under gravity alone and it is assumed that she does not hit the springboard as she descends.

(a) Find her speed when she reaches the surface of the pool

[3 marks]

(b) Find the time taken to reach the surface of the pool

[3 marks]

(c) State two physical factors which have been ignored in the model

[2 marks]

Question 2.

M1 examination question, January 2011, Q2

A ball is thrown vertically upwards with speed u ms^{-1} from a point P at height h metres above the ground. The ball hits the ground 0.75 s later.

The speed of the ball immediately before it hits the ground is 6.45 ms^{-1}

The ball is modelled as a particle.

(a) Show that $u = 0.9$

[3 marks]

(b) Find the height above P to which the ball rises before it starts to fall towards the ground again

[2 marks]

(c) Find the value of h

[3 marks]

Question 3.

M1 examination question, January 2007, Q2

A ball is projected vertically upwards with speed 21 ms^{-1} from a point *A* which is 1.5 m above the ground. After projection, the ball moves freely under gravity until it reaches the ground. Modelling the ball as a particle, find

(a) the greatest height above *A* reached by the ball

[3 marks]

(b) the speed of the ball as it reaches the ground

[2 marks]

(c) the time between the instant when the ball is projected from *A* and the instant when the ball reaches the ground

[4 marks]

Question 4.

M1 examination question, May 2010, Q6

A ball is projected vertically upwards with a speed of 14.7 ms^{-1} from a point which is 49 m above horizontal ground. Modelling the ball as a particle moving freely under gravity, find

(a) the greatest height, above the ground, reached by the ball

[4 marks]

(b) the speed with which the ball first strikes the ground

[3 marks]

(c) the total time from when the ball is projected to when it first strikes the ground

[3 marks]

Question 5.

M1 examination question, January 2008, Q2

A firework rocket starts from rest at ground level and moves vertically.
In the first 3 s of its motion, the rocket rises 27 m.
The rocket is modelled as a particle moving with constant acceleration a ms^{-2}
Find

(a) the value of a

[2 marks]

(b) the speed of the rocket 3 s after it has left the ground

[2 marks]

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity

(c) Find the height of the rocket above the ground 5 s after it has left the ground

[4 marks]

Chapter 4

Kinematics : Mechanics 1

4.1 Elaborate suvat Problems

An Investigation

A *sky spy* police camera is launched vertically upwards at 49m/s.
Use

$$s = ut + \frac{1}{2}at^2$$

to complete the left hand table showing its height at 0.5 second intervals for the first seven seconds of its flight.

$s = ?$ *find this*

$u = 49$

$v = ...$ *not relevant*

$a = -9.8$

$t = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7.$

Time, t_1	Height
0	0
0.5	23.3
1	44.1
1.5	62.5
2	78.4
2.5	91.9
3	102.9
3.5	111.5
4	
4.5	
5	
5.5	
6	
6.5	
7	

Time, t_2	Height
0	0
0.5	23.3
1	44.1
1.5	62.5
2	78.4
2.5	91.9
3	102.9
3.5	111.5
4	
4.5	
5	
5.5	

A second *sky spy* camera is accidentally launched (with the same initial velocity) from the same launcher (and so the same flight path) 1.5 seconds after the first. Complete the right hand table to show its height at 0.5 second intervals.

Identify the interval in which the two *sky spy* cameras collide.

A Mathematical Approach

In the investigation into the 'alleged incompetence' that led to the collision, it was required that the precise moment and height at which the two *sky spy* cameras collided be calculated.
(A building was damaged and a small child hurt when they fell back to Earth unable to open their parachutes).

The basic method with this sort of problem is to calculate the time first, and then find the displacement.

The first step is this;

$$u\,t_1 + \frac{1}{2}a\,t_1^{\,2} = u\,t_2 + \frac{1}{2}\,a\,t_2^{\,2}$$

But the **key** next step is this;

$$u\,t_1 + \frac{1}{2}\,a\,t_1^{2} = u\,(t_1 - 1.5) + \frac{1}{2}\,a\,(t_1 - 1.5)^2$$

Or simply;

$$u\,t + \frac{1}{2}\,a\,t^{2} = u(t - 1.5) + \frac{1}{2}\,a\,(t - 1.5)^2$$

if we keep in mind that *t* is the time into the first *sky spy* launch.

Solve the equation to calculate the moment of impact, and then the corresponding height.

4.2 Exercise

Question 1

Suppose the first *sky spy* camera had been launched at 98m/s and the second at 49m/s and 16 seconds after the first.

Calculate **(i)** The time of collision.

(ii) The height at which collision occurs.

Question 2.

Some bank robbers are driving at 17 ms^{-1}, with constant acceleration 1 ms^{-2}, as they pass a stationary police car parked in a layby.

The police start to chase them 8 seconds later, accelerating uniformly at 2 ms^{-2}.

When and how far from the layby do the police catch up with the robbers ?

Question 3.

A train passes through station A at $45ms^{-1}$, with constant acceleration $0.5\ ms^{-2}$, heading east towards station B which is 1375 metres away.

Five seconds later, another train passes through station B at a constant speed of $25\ ms^{-1}$, travelling west.

Where do the trains pass each other ?

Question 4.

A car travelling along a straight level road at a constant speed of 54 km/h passes a second car as it starts to accelerate from rest at a uniform rate of $0.5ms^{-2}$.

Find the time that elapses and the distance covered when the second car draws level with the first.

Question 5.

A ball is thrown vertically upwards from a point A with a speed of 20 ms^{-1}.
At the same instant a second ball is dropped from a point B which is 60 m vertically above A. Find the time which elapses before the two balls meet and their height above A at this instant.

Question 6.

A stone is dropped from the top of a tower.

After 1 second, another stone is thrown vertically downwards from the same point at a speed of 15 ms^{-1}.

If the stones reach the ground simultaneously, find the height of the tower.

4.3 Answers

4.3.1 Solution (4.1 Introductory Example)

Time	Height
0	0
0.5	23.3
1	44.1
1.5	62.5
2	78.4
2.5	91.9
3	102.9
3.5	111.5
4	**117.6**
4.5	**121.3**
5	**122.5**
5.5	**121.3**
6	**117.6**
6.5	**111.5**
7	**102.9**

Time	Height
0	0
0.5	23.3
1	44.1
1.5	62.5
2	78.4
2.5	91.9
3	102.9
3.5	111.5
4	**117.6**
4.5	**121.3**
5	**122.5**
5.5	**121.3**

$$u\,t + \frac{1}{2}\,a\,t^2 = u\,(t - 1.5) + \frac{1}{2}\,a\,(t - 1.5)^2$$

$$49\,t - 4.9\,t^2 = 49\,(t - 1.5) - 4.9\,(t^2 - 3t + 2.25)$$

$$49\,t - 4.9\,t^2 = 49\,t - 73.5 - 4.9\,t^2 + 14.7\,t - 11.025$$

$$0 = 14.7\,t - 84.525$$

$$t = \frac{84.525}{14.7}$$

$$t = 5.75\,s$$

$$s = ut + \frac{1}{2}\,a\,t^2$$

$$s = 49 \times 5.75 - 4.9 \times 5.75^2$$

$$s = 119.7\,m$$

4.3.2 Solutions (4.2 Exercise)

Answer 1.

$$ut + \frac{1}{2}at^2 = u(t-16) + \frac{1}{2}a(t-16)^2$$

$$98t - 4.9t^2 = 49(t-16) - 4.9(t^2 - 32t + 256)$$

$$98t - 4.9t^2 = 49t - 784 - 4.9t^2 + 156.8t - 1254.4$$

$$0 = 107.8t - 2038.4$$

$$t = \frac{2038.4}{107.8}$$

$$t = 18.91\,s$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 98 \times 18.91 - 4.9 \times 18.91^2$$

$$s = 101.0\,m$$

Answer 2.

$$17t + \frac{1}{2} \times 1 \times t^2 = 0 \times (t-8) + 0.5 \times 2 \times (t-8)^2$$

$$17t + 0.5t^2 = t^2 - 16t + 64$$

$$0 = 0.5t^2 - 33t + 64$$

$$t^2 - 66t + 128 = 0$$

$$(t - 64)(t - 2) = 0$$

$$\therefore t = 64\,s$$

(Ignore $t = 2$, as chase does not start until $t = 8$)

$$s = ut + \frac{1}{2}at^2$$

$$s = 17 \times 64 + \frac{1}{2} \times 1 \times 64^2$$

$$s = 3136\,m$$

Answer 3.
Getting the following equation is the main challenge of this question.
After that, JCA's numbers work out beautifully.
(No need to use the formula to solve quadratic equations)

$$S_A + S_B = 1375 \qquad (\textit{If you think about it!})$$

$$45t + \frac{1}{2}0.5t^2 + 25(t-5) = 1375$$

$$45t + 0.25t^2 + 25t - 125 - 1375 = 0$$

$$0.25t^2 + 70t - 1500 = 0$$

$$t^2 + 280t - 6000 = 0$$

$$(t-20)(t+300) = 0$$

$$\therefore t = 20\,s \quad \textit{Ignore negavive solution}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 45 \times 20 + \frac{1}{2} \times 0.5 \times 20^2$$

$$s = 1000\,m \; from\, Station\, A$$

(*or* 375 *m from Station B*)

Answer 4.
Watch that the km/h gets converted into m/s.
60 seconds, 900 metres

Answer 5.

$$S_A - S_B = 60 \qquad (\textit{If you think about it!})$$

$$20t + \frac{1}{2}(-9.8)t^2 - \frac{1}{2}(-9.8)t^2 = 60$$

$$20t = 60$$

$$t = 3s \qquad \therefore h = 15.9\,m \qquad above\, A$$

Answer 6.
18.49 metres

Chapter 5

Kinematics : Mechanics 1

5.1 Velocity - Time Graphs

On a Velocity - Time graph

◊ The gradient represents acceleration.

Furthermore

$$a = \frac{\Delta v}{\Delta t} \qquad \left(acceleration = \frac{change\ in\ velocity}{change\ in\ time} \right)$$

◊ Area under the graph represents distance

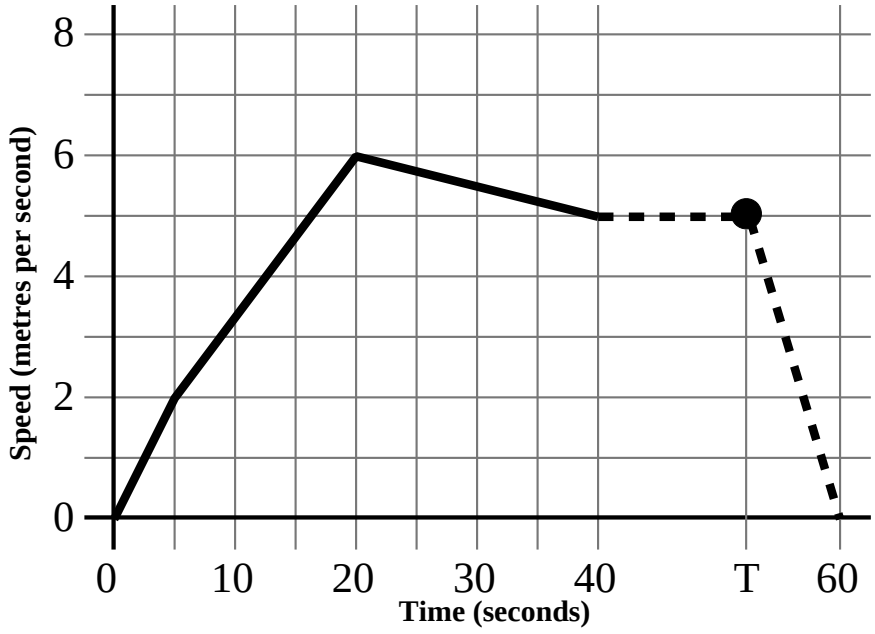
An Example

The velocity - time graph is of an orienteer's progress as she runs between two controls on a straight route.

The distance between the controls is 240 metres.

The velocity-time graph shows her

- starting from rest
- (constant) acceleration for 5 seconds to a speed of 2 ms^{-1}
- (constant) acceleration for 15 seconds to a speed of 6 ms^{-1}
- (constant) deceleration for 20 seconds to a speed of 5 ms^{-1}
- moving with constant velocity for (T - 40) seconds
- (constant) deceleration for (60 - T) seconds
- stopping
- taking 60 seconds in total



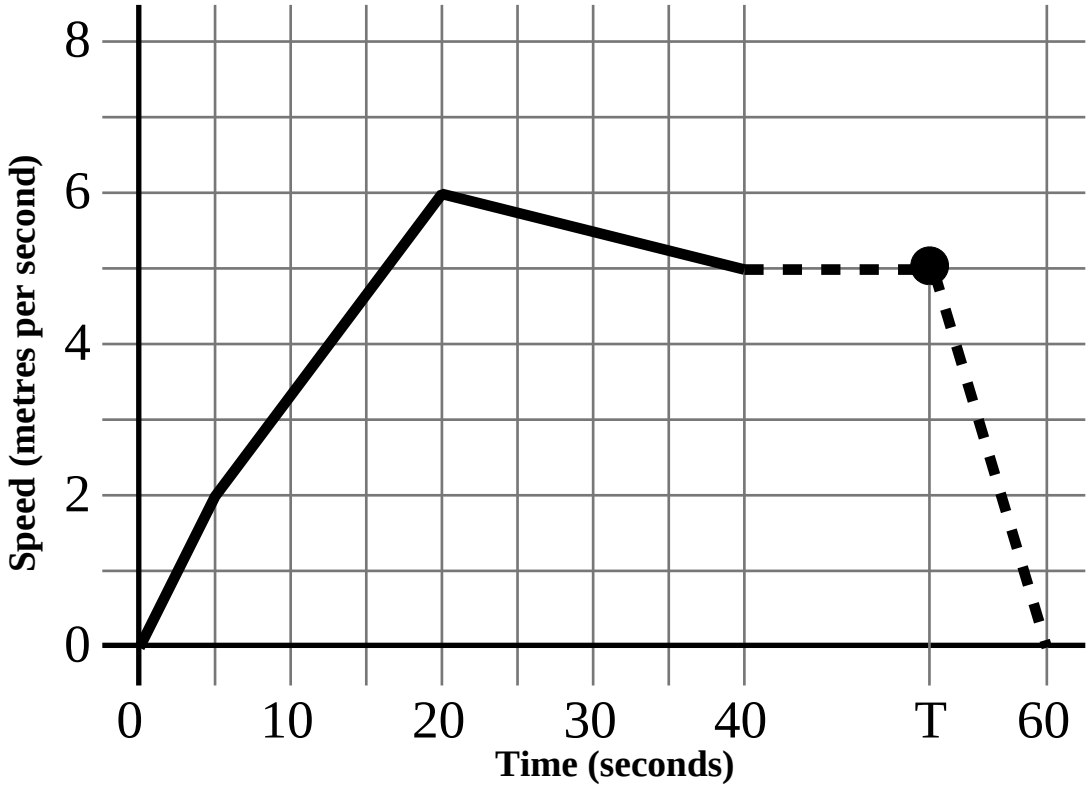
(a) What is the acceleration between $t = 5$ and $t = 20$ seconds ?

(b) Determine the value of T

Solution to The Example

(a)

(b)



5.2 Exercise

Question 1.

M1 examination question, November 2002, Q3

A car accelerates uniformly from rest to a speed of 20 ms^{-1} in T seconds. The car then travels at a constant speed of 20 ms^{-1} for $4T$ seconds and finally decelerates uniformly to rest in a further 50 s.

(a) Sketch a speed-time graph to show the motion of the car.

[2 marks]

The total distance travelled by the car is 1220 m

(b) Find the value of T

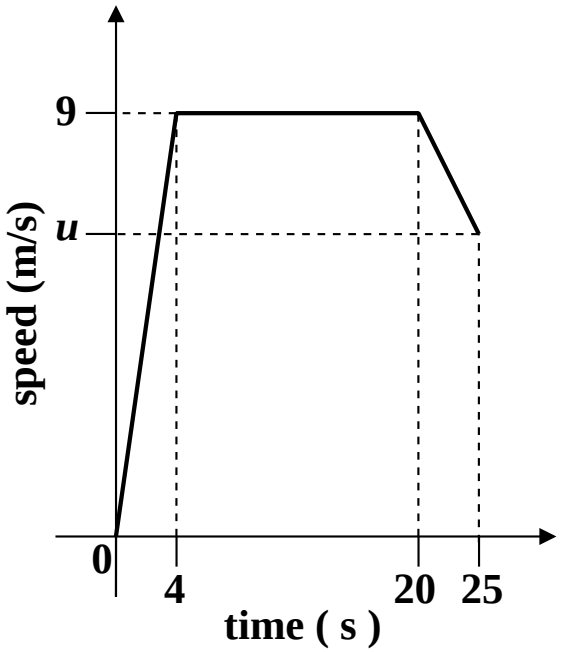
[3 marks]

(c) Find the initial acceleration of the car.

[2 marks]

Question 2.

M1 examination question, January 2005, Q3



A sprinter runs a race of 200 m.
Her total time for running the race is 25 s.
The sketch is of the speed-time graph for the motion of the sprinter.
She starts from rest and accelerates uniformly to a speed of 9 ms^{-1} in 4 s.
The speed of 9 ms^{-1} is maintained for 16 s and she then decelerates uniformly to a speed of u ms^{-1} at the end of the race.

Calculate

(a) the distance covered by the sprinter in the first 20 s of the race,

[2 marks]

(b) the value of u,

[4 marks]

(c) the deceleration of the sprinter in the last 5 s of the race

[3 marks]

Question 3.

M1 examination question, June 2005, Q5

A train is travelling at 10 ms^{-1} on a straight horizontal track.
The driver sees a red signal 135 m ahead and immediately applied the brakes.
The train immediately decelerates with constant deceleration for 12 s, reducing its speed to 3 ms^{-1}.
The driver then releases the brakes and allows the train to travel at a constant speed of 3 ms^{-1} for a further 15 s.
He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

(a) Sketch a speed-time graph to show the motion of the train

[3 marks]

(b) Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 ms^{-1}

[2 marks]

(c) Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

[5 marks]

Question 4.

M1 examination question, January 2012, Q6

A car moves along a straight horizontal road from a point A to a point B.

$AB = 885$ m

The car accelerates from rest at A to a speed of 15 ms^{-1} at a constant rate a ms^{-2}

The time for which the car accelerates is $\frac{1}{3}T$ seconds.

The car maintains the speed of 15 ms^{-1} for T seconds.

The car then decelerates at a constant rate of 2.5 ms^{-2} stopping at B.

(a) Find the time for which the car decelerates

[2 marks]

(b) Sketch a speed-time graph for the motion of the car

[2 marks]

(c) Find the value of T

[4 marks]

(d) Find the value of a

[2 marks]

(e) Sketch an acceleration-time graph for the motion of the car

[3 marks]

Question 5.

M1 examination question, January 2004, Q6

A train starts from rest at a station *A* and moves along a straight horizontal track.
For the first 10 s the train moves with constant acceleration 1.2 ms^{-2}
For the next 24 s it moves at a constant acceleration 0.75 ms^{-2}
It then moves with constant speed for *T* seconds.
Finally it slows down with constant deceleration 3 ms^{-2} until it comes to a rest at station *B*

(a) Show that, 34 s after leaving *A*, the speed of the train is 30 ms^{-1}

[3 marks]

(b) Sketch a speed-time graph to illustrate the motion of the train as it moves from *A* to *B*

[3 marks]

(c) Find the distance moved by the train during the first 34 s of its journey from A

[4 marks]

The distance from A to B is 3 km

(d) Find the value of T

[4 marks]

Chapter 6

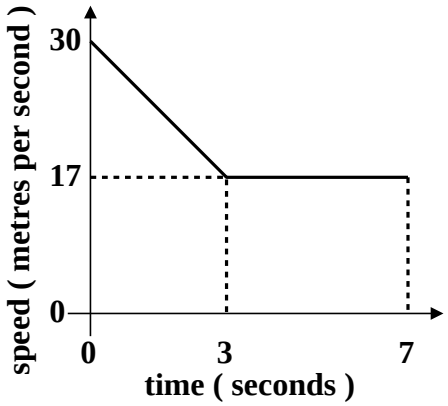
Kinematics : Mechanics 1

6.1 Topic Summary (Homework)

Whenever a numerical value of g is required, take g = 9.8 m.s^{-2}

Question 1

A car moves along a straight horizontal road.
In order to obey a speed restriction, the brakes of the car are applied for 3 seconds, reducing the car's speed from 30 m.s^{-1} to 17 m.s^{-1}.
The brakes are then released and the car continues at a constant speed of 17 m.s^{-1} for a further 4 seconds.
The speed - time graph of the car during this 7 second interval is shown below.



Find the total distance moved by the car during this 7 second interval.

[4 marks]

Question 2

A sprinter runs a 100 metre race.

He starts at a speed of 6 $m.s^{-1}$, accelerates uniformly for 2 seconds to his top speed**,** and then maintains this top speed of v $m.s^{-1}$ for the rest of the race.

He covers the whole distance of 100 metres in a total time of 11 seconds.

(i) In the space below, sketch a speed - time graph to illustrate the motion of the sprinter during the time of the race.

[3 marks]

(ii) Find the top speed, v $m.s^{-1}$, of the sprinter.

[3 marks]

Question 3
M1 examination question, June 2015, Q2

A small stone is projected vertically upwards from a point *O* with a speed of 19.6 $m.s^{-1}$

Modelling the stone as a particle moving freely under gravity,

(a) find the greatest height above *O* reached by the stone

[2 marks]

(b) find the length of time for which the stone is more than 14.7 m above *O*.

[5 marks]

Question 4

A yellow bus travels at a constant speed u for 20 seconds, and then accelerates uniformly over the next 30 seconds until it reaches a speed of 13 $m.s^{-1}$.

(i) Illustrate this information on a sketch speed - time graph.

[2 marks]

(ii) Given that the bus travels a total distance of 496 metres, find u .

[3 marks]

(iii) How long does it take the bus to travel 250 metres ?

[4 marks]

Question 5

A stone is dropped from the edge of a cliff, 320 metres above sea level.

(i) How long does it take to land in the sea ?

[2 marks]

(ii) What is its speed on impact ?

[3 marks]

Suppose another stone was thrown downwards at 25 $m.s^{-1}$ from the cliff edge, two seconds after the first stone was released.

Further suppose that his second stone catches up, and collides with, the first stone.

(iii) How long after this throw would it take for the stones to collide ?

[5 marks]

(iv) At what height *above the sea* would the two stones be when they collided ?

[2 marks]

(v) Do you consider your part **(ii)** answer to be greater than or less than the answer that might be obtained in reality ?
Justify your answer.

[2 marks]