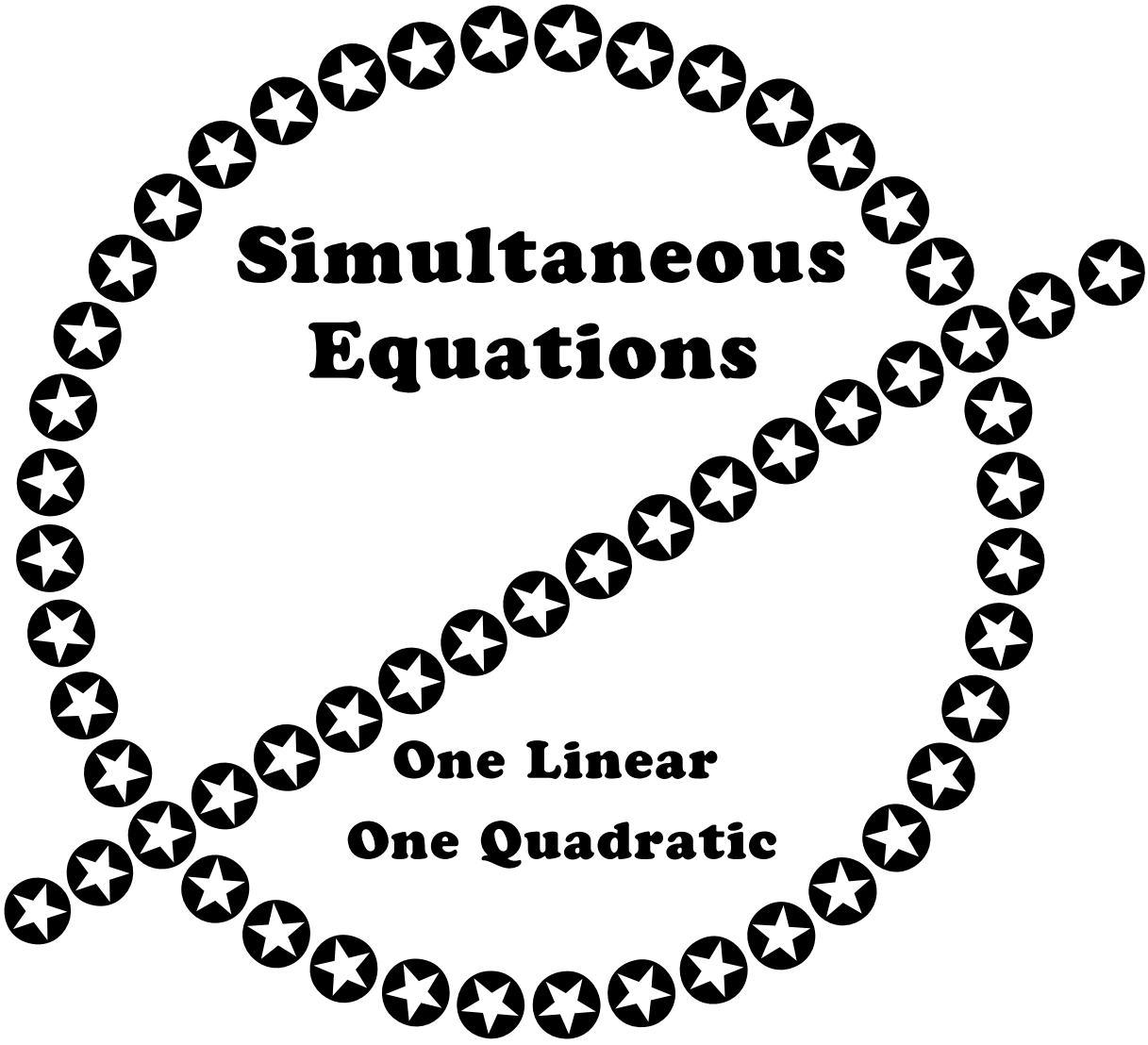
Simultaneous
Equations
One Linear
One Quadratic

Simultaneous Equations
One Linear and One Quadratic

Chapter 1

1.1 Introduction

Previously, we have looked at solving simultaneous equations where both equations were linear. That is, they were *line like*. Geometrically, this equivalent to finding the crossing point of two straight lines.

Now we move on to the situation where one of the two equations is quadratic.
This could be a circle, for example;

$$x^2 + y^2 = 64$$

or a parabola, for example;

$$y = x^2 + 2x + 1$$

Here is an example of an examination question;

GCSE, June 2011, paper 3H, Q22
Solve the simultaneous equations

$$y = 2x - 3 \qquad (\,line\,)$$

$$x^2 + y^2 = 2 \qquad (\,circle\,)$$

Geometrically, this is asking for the points (if any) where the line cuts the circle.
Typically there will be two points but there could be one or none.

This topic is really five previous topics glued together.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

We'll revise each of these separately, then look at how to glue the bits together.

In an examination, a *solve these simultaneous equations question* is worth about six marks. The bits glued together will also be tested separately elsewhere in the exam. So, being good at this topic is potentially worth about 6% in each of the two papers.

1.2 FOIL

This is a first and crucial step.
If you get this wrong, the mark scheme often states that no marks are to be awarded.
So an error here really is, *Do NOT pass Go, Do not collect £200*

Questions.
Expand the brackets;

(i) $(x + 6)(x + 5)$

(ii) $(x + 1)(4x + 7)$

(iii) $(2x + 5)(3x + 4)$

(iv) $(8x + 5)(11x + 9)$

(v) $(x + 8)(x - 3)$

(vi) $(x - 5)(3x + 4)$

(vii) $(x - 7)(5x - 2)$

(viii) $(x + 5)^2$

(ix) $(2x + 7)^2$

(x) $(5x - 3)^2$

The last three questions are the most important.

1.3 Gathering together like terms

Example
Simplify $3x^2 - 3 + 4x + 2x + x^2 + 7$
Try this, then check with the answer at the bottom of the following page.

Questions.
Simplify each of the following;

(i) $x^2 + 5x + 7x + 11$

(ii) $2x^2 + 11x - 6x + 3$

(iii) $5x^2 + 4x + 3 + 5x - 2$

(iv) $x^2 + 4x - 7 - 9x + 3$

(v) $9x^2 + 13x - 6x^2 + 3 - x$

(vi) $3x^2 + 14x + (5x - 3) - x^2$

(vii) $5x^2 - 7x - (4x + 3)$

(viii) $x^2 + 7x + (7 - 5x) - 3$

(ix) $8x^2 + 14x - (4x^2 - 3x) - x^2$

(x) $4(3x^2 + 2x) - (7 - 5x) - 1$

1.4 Rearranging equations into the form $f(x) = 0$

Example

Rearrange the following equation into the form $f(x) = 0$

$5x^2 + 4x + 8 = 2x + 5$

Try this, then check with the answer at the bottom of the following page.

Questions.

Rearrange each of the following equation into the form $f(x) = 0$

(i) $8x^2 + 9x + 5 = 4x + 2$

(ii) $2x^2 + 3x + 11 = 7x + 3$

(iii) $7x^2 - 5x + 1 = 3x - 7$

(iv) $9x^2 + 7x - 8 = 5 - 3x$

Previous page answer: $4x^2 + 6x + 4$

(v) $8x^2 + 14x - 8 = 2x^2 - 3x + 2$

(vi) $2x^2 + 15x - 4 = 2x^2 + 3x + 28$

(vii) $5x^2 + (6x - 5) = 9 - 2x + x^2$

(viii) $11x^2 - (4x + 3) = 5x^2 - 15$

(ix) $8x^2 - (7x - 6) = 5 - 3x$

(x) $7x^2 - (3 + 7x) = 2x^2 - (6x - 1)$

1.5 Mixture

Before tackling the last two items on the introduction's lists, here is some question that mix the first three ideas.

Questions.
Rearrange each of the following equation into the form $f(x) = 0$

(i) $(x + 4)(x + 6) = 5x + 3$

(ii) $(3x + 9)(2x + 3) = 8x - 13$

(iii) $(x + 5)(x - 5) = 7x + 13$

(iv) $(x + 4)(3x - 5) = 2x^2 - 5$

(v) $(x + 7)^2 = 4x + 7$

(vi) $(2x - 3)^2 = (x + 2)^2$

Previous page answer: $5x^2 + 2x + 3 = 0$

1.6 Answers

1.6.1 Solutions (1.2 FOIL)

(i) $x^2 + 11x + 30$ **(ii)** $4x^2 + 11x + 7$

(iii) $6x^2 + 23x + 20$ **(iv)** $88x^2 + 127x + 45$

(v) $x^2 + 5x - 24$ **(vi)** $3x^2 - 11x - 20$

(vii) $5x^2 - 37x + 14$ **(viii)** $x^2 + 10x + 25$

(ix) $4x^2 + 28x + 49$ **(x)** $25x^2 - 30x + 9$

1.6.2 Solutions (1.3 Gathering together like terms)

(i) $x^2 + 12x + 11$ **(ii)** $2x^2 + 5x + 3$

(iii) $5x^2 + 9x + 1$ **(iv)** $x^2 - 5x - 4$

(v) $3x^2 + 12x + 3$ **(vi)** $2x^2 + 19x - 3$

(vii) $5x^2 - 11x - 3$ **(viii)** $x^2 + 2x + 4$

(ix) $3x^2 + 17x$ **(x)** $12x^2 + 13x - 8$

1.6.3 Solutions (1.4 Rearranging equations into the form $f(x) = 0$)

(i) $8x^2 + 5x + 3 = 0$ **(ii)** $2x^2 - 4x + 8 = 0$

(iii) $7x^2 - 8x + 8 = 0$ **(iv)** $9x^2 + 10x - 13 = 0$

(v) $6x^2 + 17x - 10 = 0$ **(vi)** $12x - 32 = 0$

(vii) $4x^2 + 8x - 14 = 0$ **(viii)** $6x^2 - 4x + 12 = 0$

(ix) $8x^2 - 4x + 1 = 0$ **(x)** $5x^2 - x - 4 = 0$

1.6.4 Solutions (1.5 Mixture)

(i) $x^2 + 5x + 21 = 0$ **(ii)** $6x^2 + 19x + 40 = 0$

(iii) $x^2 - 7x - 38 = 0$ **(iv)** $x^2 + 7x - 15 = 0$

(v) $x^2 + 10x + 42 = 0$ **(vi)** $3x^2 - 16x + 5 = 0$

Chapter 2

Simultaneous Equations : GCSE

2.1 Recap

To solve questions like the following;

GCSE, June 2011, paper 3H, Q22
Solve the simultaneous equations

$$y = 2x - 3 \qquad (line)$$

$$x^2 + y^2 = 2 \qquad (circle)$$

We need to be competent at the following five topics;

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

2.2 Revision Exercise

Question 1.

- Expanding brackets, FOIL

Expand the brackets;

(i) $(x + 6)(x + 11)$

(ii) $(3x + 5)(4x - 3)$

(iii) $(x + 4)^2$

Question 2.

- Gathering together like terms

Simplify each of the following;

(i) $x^2 + 9x + 17x + 18$

(ii) $7x^2 + 17x - 3x^2 + 13 - 5x$

(iii) $15x^2 - (6x - 7) + 6x^2 - 15$

Question 3.

- Rearranging equations into the form $f(x) = 0$

Rearrange each of the following equation into the form $f(x) = 0$

(i) $19x^2 + 2x + 6 = 5x + 3$

(ii) $5x^2 + 12x - 8 = 2x^2 - 7x + 8$

(iii) $13x^2 - (7x - 6) = 1 - 4x$

Question 4.

This question is a mix the first three ideas.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$

Rearrange each of the following equation into the form $f(x) = 0$

(i) $(x + 5)(x + 8) = 4x + 3$

(ii) $(x + 5)^2 = 7x + 9$

(iii) $(3x - 2)^2 = (x + 1)^2$

2.3 Answers

2.3.1 Solutions (2.2 Revision Exercise)

Answer 1.

(i) $x^2 + 17x + 66$ **(ii)** $12x^2 + 11x - 15$

(iii) $x^2 + 8x + 16$

Answer 2.

(i) $x^2 + 26x + 18$ **(ii)** $4x^2 + 12x + 13$

(iii) $21x^2 - 6x - 8$

Answer 3.

(i) $19x^2 - 3x + 3 = 0$ **(ii)** $3x^2 + 19x - 16 = 0$

(iii) $13x^2 - 3x + 5 = 0$

Answer 4.

(i) $x^2 + 9x + 37 = 0$ **(ii)** $x^2 + 3x + 16 = 0$

(iii) $8x^2 - 14x + 3 = 0$

Chapter 3

Simultaneous Equations : GCSE

3.1 Factorising Quadratics

The word *factorise* means *make brackets.*

When asked to factorise a quadratic such as, $x^2 + 11x + 24$, we need to find an equivalent expression of the form $(x + \alpha)(x + \beta)$ where α and β are two numbers that we must work out.

Example
Factorise, $x^2 + 11x + 24$

Example
Factorise, $x^2 + 3x - 10$

3.2 Exercise

Question 1.
Factorise;

(i) $x^2 + 10x + 21$ **(ii)** $x^2 + 11x + 30$

(iii) $x^2 + 9x + 14$ **(iv)** $x^2 + 8x + 15$

(v) $x^2 + 14x + 33$ **(vi)** $x^2 + 6x + 9$

(vii) $x^2 + 10x + 9$

(viii) $x^2 + 14x + 13$

(ix) $x^2 + 14x + 48$

(x) $x^2 + 18x + 77$

Question 2.
Factorise;

(i) $x^2 + 2x - 3$

(ii) $x^2 + 5x - 14$

(iii) $x^2 + 9x - 22$

(iv) $x^2 + 2x - 15$

(v) $x^2 - 2x - 15$

(vi) $x^2 - 4x - 21$

(vii) $x^2 - 8x - 20$

(viii) $x^2 - 8x - 33$

(ix) $x^2 - 3x - 40$

(x) $x^2 - 6x - 40$

Question 3.
Factorise;

(i) $x^2 - 5x + 6$ **(ii)** $x^2 - 8x + 15$

(iii) $x^2 - 10x + 21$ **(iv)** $x^2 - 9x + 20$

(v) $x^2 - 10x + 25$ **(vi)** $x^2 - 7x + 6$

(vii) $x^2 - 10x + 16$ **(viii)** $x^2 - 8x + 12$

(ix) $x^2 - 15x + 44$ **(x)** $x^2 - 14x + 49$

Question 4.
Factorise;

(i) $x^2 + 15x + 50$ **(ii)** $x^2 + 5x - 50$

(iii) $x^2 - 15x + 50$ **(iv)** $x^2 - 5x - 50$

3.3 Answers

3.3.1 Solutions (3.2 Exercise)

Answer 1.

(i) $(x+3)(x+7)$ **(ii)** $(x+5)(x+6)$

(iii) $(x+2)(x+7)$ **(iv)** $(x+3)(x+5)$

(v) $(x+3)(x+11)$ **(vi)** $(x+3)^2$

(vii) $(x+1)(x+9)$ **(viii)** $(x+1)(x+13)$

(ix) $(x+6)(x+8)$ **(x)** $(x+7)(x+11)$

Answer 2.

(i) $(x+3)(x-1)$ **(ii)** $(x+7)(x-2)$

(iii) $(x+11)(x-2)$ **(iv)** $(x+5)(x-3)$

(v) $(x-5)(x+3)$ **(vi)** $(x-7)(x+3)$

(vii) $(x-10)(x+2)$ **(viii)** $(x-11)(x+3)$

(ix) $(x-8)(x+5)$ **(x)** $(x-10)(x+4)$

Answer 3.

(i) $(x-2)(x-3)$ **(ii)** $(x-3)(x-5)$

(iii) $(x-3)(x-7)$ **(iv)** $(x-4)(x-5)$

(v) $(x-5)^2$ **(vi)** $(x-6)(x-1)$

(vii) $(x-8)(x-2)$ **(viii)** $(x-6)(x-2)$

(ix) $(x-4)(x-11)$ **(x)** $(x-7)^2$

Answer 4.

(i) $(x+5)(x+10)$ **(ii)** $(x+10)(x-5)$

(iii) $(x-5)(x-10)$ **(iv)** $(x-10)(x+5)$

Chapter 4

Simultaneous Equations : GCSE

4.1 Solving Quadratic Equations

Example

Solve; $x^2 + 6x + 8 = 0$

4.2 Exercise

Solve these quadratic equations;

(i) $x^2 + 8x + 15 = 0$

(ii) $x^2 + 5x + 4 = 0$

(iii) $x^2 + 9x + 14 = 0$

(iv) $x^2 + 9x + 18 = 0$

(v) $x^2 + 15x + 44 = 0$

(vi) $x^2 + 11x + 24 = 0$

(vii) $x^2 + 11x + 30 = 0$

(viii) $x^2 + 12x + 27 = 0$

(ix) $x^2 + 15x + 50 = 0$

(x) $x^2 + 12x + 32 = 0$

Example

Solve; $x^2 - 6x - 55 = 0$

4.3 Exercise

Solve these quadratic equations;

(i) $x^2 - 5x - 14 = 0$

(ii) $x^2 - 4x - 21 = 0$

(iii) $x^2 - 10x - 24 = 0$

(iv) $x^2 - x - 30 = 0$

(v) $x^2 - 6x - 40 = 0$ **(vi)** $x^2 + 6x - 16 = 0$

(vii) $x^2 + 8x - 33 = 0$ **(viii)** $x^2 + 5x - 50 = 0$

(ix) $x^2 + 16x - 17 = 0$ **(x)** $x^2 + 3x - 40 = 0$

Harder Question

(xi) $2x^2 + 32x - 34 = 0$ **HINT** : *Divide throughout by 2, first.*

Harder Question

(xii) $3x^2 + 9x - 120 = 0$ **HINT** : *Divide throughout by 3, first.*

4.3 Answers

4.3.1 Solutions (Example)

$$x^2 + 6x + 8 = 0$$

$$(x + 4)(x + 2) = 0$$

EITHER $x + 4 = 0$ *OR* $x + 2 = 0$

$$\therefore \; x = -4, \; -2 \qquad (\textit{two solutions})$$

4.3.2 Solutions (4.2 Exercise)

(i) -5, -3 **(ii)** -4, -1

(iii) -7, -2 **(iv)** -6, -3

(v) -11, -4 **(vi)** -8, -3

(vii) -6, -5 **(viii)** -9, -3

(ix) -10, -5 **(x)** -8, -4

4.3.3 Solutions (Example)

$$x^2 - 6x - 55 = 0$$

$$(x + 5)(x - 11) = 0$$

EITHER $x + 5 = 0$ *OR* $x - 11 = 0$

$$\therefore \; x = -5, \; 11 \qquad (\textit{two solutions})$$

4.3.4 Solutions (4.3 Exercise)

(i) -2, 7 **(ii)** -3, 7

(iii) -2, 12 **(iv)** -5, 6

(v) -4, 10 **(vi)** -8, 2

(vii) -11, 3 **(viii)** -10, 5

(ix) 1, -17 **(x)** -8, 5

Harder Questions

(xi) 1, -17 **(xii)** -8, 5

Chapter 5

Simultaneous Equations : GCSE

5.1 Examination Style Questions

Example

Question: Solve the simultaneous equations

$$y = x^2$$
$$y = 5x - 6$$

Solution: Using *the method of substitution.*

$$x^2 = 5x - 6$$

- Rearranging equations into the form $f(x) = 0$

$$x^2 - 5x + 6 = 0$$

- Factorising quadratics

$$(x - 2)(x - 3) = 0$$

- Solving quadratic equations

$$EITHER \quad x - 2 = 0 \quad OR \quad x - 3 = 0$$
$$x = 2 \qquad x = 3$$

The answer is points where the line intersects the parabola. Use the equation of the line $y = 5x - 6$ to work out y when x is 2, and 3.

$$SOLUTIONS : (2, 4) \; and \; (3, 9)$$

5.2 Exercise

Use the method of substitution to obtain a quadratic equation in only one variable. Solve your equation, and find the possible pairs of values for x and y.

(i) $y = x^2$
$y = 8x - 12$

(ii) $y = x^2$
$y = 11x - 28$

(iii) $y = x^2$
$y = 2x + 24$

(iv) $y = x^2 + 10$
$y = 4 - 7x$

(v) $y = x^2 - 14$
$y = 2x + 21$

(vi) $y = x^2 + 3$
$y = 30 - 6x$

(vii) $y = x^2 + 2x$
$y = 5x + 28$

(viii) $y = x^2 - 4x + 2$
$y = 7x - 8$

(ix) $y = x^2 + 3x - 10$
$y = 4x + 20$

(x) $y = x^2$
$y = 7x - 12$

5.3 Examination Question

GCSE, November 2006, paper 3H, Q18

Solve the simultaneous equations

$$y = x^2$$

$$y = 2x + 15$$

[5 marks]

5.4 Homework

Use the method of substitution to obtain a quadratic equation in only one variable. Solve your equation, and find the possible pairs of values for x and y.

(i) $y = x^2$
$y = 10 - 3x$

(ii) $y = x^2$
$y = 8 - 7x$

(iii) $y = x^2$
$y = 5x + 36$

(iv) $y = x^2 + 20$
$y = 17x - 50$

(v) $y = x^2 - 2x + 1$
$y = 10x - 31$

(vi) $y = x^2 + 4x$
$y = 6x + 48$

(vii) $y = x^2 - 4x + 5$
$y = 3 - x$

(viii) $y = x^2 + 5x + 17$
$y = 2 - 3x$

5.5 Answers

5.5.1 Solutions (5.2 Exercise)

(i) $x^2 - 8x + 12 = 0$
$(x - 6)(x - 2) = 0$
POINTS : (2, 4) & (6, 36)

(ii) $x^2 - 11x + 28 = 0$
$(x - 4)(x - 7) = 0$
POINTS : (4, 16) & (7, 49)

(iii) $x^2 - 2x - 24 = 0$
$(x - 6)(x + 4) = 0$
POINTS : (6, 36) & (-4, 16)

(iv) $x^2 + 7x + 6 = 0$
$(x + 1)(x + 6) = 0$
POINTS : (-1, 11) & (-6, 46)

(v) $x^2 - 2x - 35 = 0$
$(x - 7)(x + 5) = 0$
POINTS : (7, 35) & (-5, 11)

(vi) $x^2 + 6x - 27 = 0$
$(x + 9)(x - 3) = 0$
POINTS : (-9, 84) & (3, 12)

(vii) $x^2 - 3x - 28 = 0$
$(x - 7)(x + 4) = 0$
POINTS : (7, 63) & (-4, 8)

(viii) $x^2 - 11x + 10 = 0$
$(x - 10)(x - 1) = 0$
POINTS : (10, 62) & (1, -1)

(ix) $x^2 - x - 30 = 0$
$(x - 6)(x + 5) = 0$
POINTS : (6, 44) & (-5, 0)

(x) $x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
POINTS : (3, 9) & (4, 16)

5.5.2 Solutions (5.3 Examination Question)

$x^2 - 2x - 15 = 0$
$(x - 5)(x + 3) = 0$
POINTS : (5, 25) & (-3, 9)

5.5.3 Solutions (5.4 homework)

(i) $x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$
POINTS : (-5, 25) & (2, 4)

(ii) $x^2 + 7x - 8 = 0$
$(x + 8)(x - 1) = 0$
POINTS : (-8, 64) & (1, 1)

(iii) $x^2 - 5x - 36 = 0$
$(x - 9)(x + 4) = 0$
POINTS : (9, 81) & (-4, 16)

(iv) $x^2 - 17x + 70 = 0$
$(x - 10)(x - 7) = 0$
POINTS : (10, 120) & (7, 69)

(v) $x^2 - 12x + 32 = 0$
$(x - 4)(x - 8) = 0$
POINTS : (4, 9) & (8, 49)

(vi) $x^2 - 2x - 48 = 0$
$(x - 8)(x + 6) = 0$
POINTS : (8, 96) & (-6, 12)

(vii) $x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
POINTS : (1, 2) & (2, 1)

(viii) $x^2 + 8x + 15 = 0$
$(x + 3)(x + 5) = 0$
POINTS : (-3,, 11) & (-5, 17)

Chapter 6

Simultaneous Equations : GCSE

6.1 Exam Bits

This is a collection of examination questions that involve the five topics we've been looking at. Often a question is testing just one of the five.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

6.2 Exercise

Question 1.
GCSE, May 2008, paper 4H, Q1
(a) Solve

$$6x + 13 = 2x + 7$$

[3 marks]

(b) Solve

$$\frac{y}{5} - 2 = 4$$

[2 marks]

Question 2.
GCSE, November 2006, paper 3H, Q7
Solve the inequality

$$9x - 2 < 5x + 4$$

[3 marks]

Question 3.
GCSE, June 2010, paper 4H, Q4

(a) Multiply out

$$5\,(n + 6)$$

[1 marks]

(b) Simplify

$$y \times y \times y \times y \times y \times y$$

[1 marks]

(c) Solve

$$4\,(x - 2) = 3$$

[3 marks]

Question 4.
GCSE, May 2007, paper 3H, Q6

(a) Expand and simplify

$$3\,(4x - 5) - 4\,(2x + 1)$$

[2 marks]

(b) Expand and simplify

$$(y + 8)\,(y + 3)$$

[2 marks]

(c) Expand

$$p\,(5p^2 + 4)$$

[2 marks]

Question 5.
GCSE, May 2007, paper 3H, Q9
(a) Solve

$$5x - 4 = 2x + 7$$

[2 marks]

(b) Solve

$$\frac{7 - 2y}{4} = 2y + 3$$

[4 marks]

Question 6.
GCSE, November 2010, paper 4H, Q20

Solve the simultaneous equations

$$y = x^2$$
$$y = 7x - 10$$

[5 marks]

Question 7.
GCSE, June 2010, paper 4H, Q1
Solve

$$6y - 9 = 3y + 7$$

[3 marks]

Question 8.
GCSE, June 2010, paper 4H, Q13(a)
Solve

$$x^2 - 8x + 12 = 0$$

[3 marks]

Question 9.
GCSE, November 2010, paper 3H, Q13(a)
Factorise

$$x^2 - 8x + 15$$

[2 marks]

Question 10.
GCSE, June 2009, paper 4H, Q16
(a) Factorise

$$2x^2 - x - 3$$

[2 marks]

(b) Hence write down the solutions of

$$2x^2 - x - 3 = 0$$

[1 mark]

Question 11.
GCSE, November 2009, paper 3H, Q2
Solve

$$8y - 9 = 5y + 3$$

[3 marks]

Question 12.
GCSE, November 2009, paper 3H, Q9
(a) Expand and simplify fully

$$2(w - 3) + 3(w + 5)$$

[2 marks]

(b) Solve the equation

$$\frac{x + 5}{3} = 9$$

[2 marks]

(c) Solve the inequality

$$5y + 7 < 13$$

[2 marks]

Question 13.
GCSE, November 2009, paper 4H, Q12(a)
Expand and simplify

$$(p + 7)(p - 4)$$

[2 marks]

Question 14.
GCSE, November 2009, paper 4H, Q2

(a) Factorise

$$n^2 - 4n$$

[2 marks]

(b) Solve

$$8 - 5x = 2$$

[3 marks]

Question 15.
GCSE, May 2009, paper 3H, Q5

(a) Factorise

$$p^2 + 7p$$

[2 marks]

(b) Solve

$$4 - 5x = 2$$

[3 marks]

(c) Simplify

$$t^3 \times t^6$$

[1 mark]

(d) Expand and simplify

$$3(4y + 5) - 5(2y + 3)$$

[2 marks]

Question 16.
GCSE, November 2008, paper 4H, Q17(a)
Factorise

$$2x^2 + 5x + 3$$

[2 marks]

Question 17.
GCSE, November 2008, paper 4H, Q6
(a) Multiply out

$$5(x - 2)$$

[2 marks]

(b) Solve the equation

$$\frac{x}{4} + 3 = 10$$

[2 marks]

(c) Solve the inequality

$$5x - 6 > 2$$

[2 marks]

Question 18.
GCSE, November 2008, paper 4H, Q6
Solve

$$5(x - 4) = 35$$

[3 marks]

Question 19.
GCSE, November 2007, paper 4H, Q2
(a) Factorise

$$5x - 20$$

[1 mark]

(b) Factorise

$$y^2 + 6y$$

[2 marks]

Question 20.
GCSE, May 2006, paper 3H, Q13(a)
Expand and simplify

$$(3x - 5)(4x + 7)$$

[2 marks]

Question 21.
GCSE, May 2006, paper 4H, Q2
(a) Factorise

$$3x^2 - 2x$$

[1 mark]

(b) Expand

$$y^3(y - 4)$$

[2 marks]

(c) Here is a formula used in physics

$$v = u + at$$

Find the value of t when $v = 30$, $u = 5$ and $a = 10$

[2 marks]

Question 22.
GCSE, May 2006, paper 4H, Q8
(a) Solve

$$3(x + 4) = 27$$

[3 marks]

(b) Solve

$$y^2 - 2y - 120 = 0$$

[3 marks]

Question 23.
GCSE, May 2006, paper 4H, Q12(a)
Factorise

$$3x^2 - 13x + 4$$

[2 marks]

Question 24.
GCSE, May 2006, paper 4H, Q17

Solve the simultaneous equations;

$$y = 2x + 1$$
$$x^2 + y^2 = 13$$

[6 marks }

6.3 Answers

6.3.1 Solutions (6.2 Exercise)

Answer 1.
(a) $x = -1.5$
(b) $y = 30$

Answer 2.
$x < 1.5$

Answer 3.
(a) $5n + 30$
(b) y^6
(c) 2.75

Answer 4.
(a) $4x - 19$
(b) $y^2 + 11y + 24$
(c) $5p^3 + 4p$

Answer 5.
(a) $3\frac{2}{3}$
(b) -0.5

Answer 6.
(2, 4) (5, 25)

Answer 7.
$5\frac{1}{3}$

Answer 8.
$x = 2$ $x = 6$

Answer 9.
$(x - 3) (x - 5)$

Answer 10.
(a) $(2x - 3) (x + 1)$
(b) $x = -1$, $x = \frac{3}{2}$

Answer 11.
$y = 4$

Answer 12.
(a) $5w + 9$
(b) $x = 22$
(c) $y < 1.2$

Answer 13.
$p^2 + 3p - 28$

Answer 14.
(a) $n (n - 4)$
(b) $x = 1.2$

Answer 15.
(a) $p (p + 7)$
(b) $x = 0.4$
(c) t^9
(d) $2y$

Answer 16.
$(2x + 3) (x + 1)$

Answer 17.
(a) $5x - 10$
(b) $x = 28$
(c) $x > 1.6$

Answer 18.
$x = 11$

Answer 19.
(a) $5 (x - 4)$
(b) $y (y + 6)$

Answer 20.
$12 x^2 + x - 35$

Answer 21.
(a) $x (3x - 2)$
(b) $y^4 - 4 y^3$
(c) $t = 2.5$

Answer 22.
(a) $x = 5$
(b) $y = -10, y = 12$

Answer 23.
$(3x - 1) (x - 4)$

Answer 24
(-2, -3) (1.2, 3.4)

Chapter 7

Simultaneous Equations : GCSE

7.1 T E S T

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

You may use a calculator
75 marks in total are available

Question 1.

Solve

(i)

$$5x + 13 = -27$$

[2 marks]

(ii)

$$\frac{x}{4} = 0.625$$

[2 marks]

(iii)

$$\frac{3x}{7} + 8 = 14$$

[2 marks]

(iv)

$$4(x + 7) = 44$$

[2 marks]

(v)

$$x^2 = 25$$

[2 marks]

(vi)

$$\sqrt{x} + 9 = 5$$

[2 marks]

Question 2.
GCSE, June 2011, paper 3H, Q3

(a) Factorise

$$w^2 - 9w$$

[2 marks]

(b) Solve

$$5x - 1 = 2x - 7$$

[3 marks]

(c) Expand and simplify

$$(y - 7)(y + 3)$$

[2 marks]

Question 3.
GCSE, June 2011, paper 4H, Q5

(a) Simplify

(i)

$$a \times a \times a \times a$$

[1 mark]

(ii)

$$5a \times 6b$$

[1 mark]

(iii)

$$q^8 \times q^2$$

[1 mark }

(b) Solve

$$5 - 2y = 12$$

[2 marks]

(c)

$$v = w^2 - 2w$$

Work out the value of v when $w = 6$.

[2 marks]

Question 4.
GCSE, May 2008, paper 3H, Q9

(a) Solve

$$7(x - 1) = 5 - 2x$$

[3 marks]

(b) **(i)** Solve the inequality

$$4x + 5 \leqslant 21$$

[2 marks]

(ii) n is a positive integer.
Write down all the values of n which satisfy

$$4n + 5 \leqslant 21$$

[2 marks]

Question 5.
GCSE, November 2007, paper 3H, Q9

(a) Expand

$$4(v + 3)$$

[1 mark]

(b) Simplify

$$\frac{w^3 \times w^7}{w^2}$$

[2 marks]

(c) Solve the equation

$$\frac{17 - x}{7} = 3$$

[3 marks]

(d) Solve the inequality

$$4y - 5 < 6$$

[2 marks]

Question 6.
GCSE, May 2008, paper 3H, Q14

(a) Factorise

$$10y - 15$$

[1 mark]

(b) Factorise completely

$$9p^2q + 12pq^2$$

[2 marks]

(c) **(i)** Factorise

$$x^2 + 6x - 16$$

[2 marks]

(ii) Solve

$$x^2 + 6x - 16 = 0$$

[1 mark]

Question 7.
GCSE, June 2011, paper 4H, Q12

(i) Solve the inequality

$$2x + 13 \geqslant 6$$

[2 marks]

(ii) n is a **negative** integer.
Write down all the values of n which satisfy

$$2n + 13 \geqslant 6$$

[2 marks]

Question 8.
Expand the brackets and simplify

$$(x - 5)(x + 7)$$

[2 marks]

Question 9.
Expand the brackets and simplify

$$(3x + 5)(2x + 1)$$

[2 marks]

Question 10.
GCSE, November 2007, paper 3H, Q13 (b) (c)
Factorise

(a)

$$x^2 - x - 12$$

[2 marks]

(b)

$$3x^2 + 7x + 2$$

[2 marks }

Question 11.
Solve the simultaneous equations

$$y = x^2$$
$$y = 5x + 36$$

[5 marks]

Question 12.
GCSE, June 2011, paper 4H, Q15 (a)
Simplify

$$\frac{8\,(x-3)^2}{4\,(x-3)}$$

[2 marks]

Question 13.
GCSE, November 2007, paper 3H, Q16 (a)
Simplify

$$\frac{x^2-3x}{2x-6}$$

HINT : make brackets

[3 marks]

Question 14.
GCSE, June 2011, paper 3H, Q22
Solve the simultaneous equations

$$y = 2x - 3$$
$$x^2 + y^2 = 2$$

[6 marks]

7.2 Answers

7.2.1 Solutions (7.1 Test)

Answer 1.

(i) $x = -8$
(ii) $x = 2.5$
(iii) $x = 14$
(iv) $x = 4$
(v) $x = -5, 5$
(vi) $x = 16$

Answer 2.

(a) $w (w - 9)$
(b) $x = -2$
(c) $y^2 - 4y - 21$

Answer 3.

(a) **(i)** a^4
(ii) $30ab$
(iii) q^{10}
(b) $y = -3.5$
(c) $v = 24$

Answer 4.

(a) $x = \frac{4}{3}$
(b) **(i)** $x \leqslant 4$
(ii) 1, 2, 3, 4

Answer 5.

(a) $4v + 12$
(b) w^8
(c) $x = -4$
(d) $y < 2.75$

Answer 6.

(a) $5 (2y - 3)$
(b) $3pq (3p + 4q)$
(c) **(i)** $(x + 8) (x - 2)$
(ii) $x = -8, 2$

Answer 7.

(i) $x \geqslant -3.5$
(ii) -3, -2, -1

Answer 8.

$x^2 + 2x - 35$

Answer 9.

$6x^2 + 13x + 5$

Answer 10.

(a) $(x - 4) (x + 3)$
(b) $(3x + 1) (x + 2)$

Answer 11.

$x^2 - 5x - 36 = 0$
gives (9, 81) & (-4, 16)

Answer 12.

$2 (x - 3)$

Answer 13.

$\frac{x}{2}$

Answer 14.

$5x^2 - 12x + 7 = 0$
gives (1.4, -0.2) & (1, -1)

Chapter 8

8.1 The Tough Examination Question Solution

Here is the last question from the Chapter 7 test;

GCSE, June 2011, paper 3H, Q22
Solve the simultaneous equations

$$y = 2x - 3 \qquad (line)$$
$$x^2 + y^2 = 2 \qquad (circle)$$

We begin by using *the method of substitution.*

$$x^2 + y^2 = 2$$
$$x^2 + (2x - 3)^2 = 2$$

- Expanding brackets, FOIL

$$x^2 + (2x - 3)(2x - 3) = 2$$
$$x^2 + 4x^2 - 6x - 6x + 9 = 2$$

- Gathering together like terms

$$5x^2 - 12x + 9 = 2$$

- Rearranging equations into the form $f(x) = 0$

$$5x^2 - 12x + 7 = 0$$

- Factorising quadratics

$$(5x - 7)(x - 1) = 0$$

- Solving quadratic equations

$$EITHER\ 5x - 7 = 0 \quad OR \quad x - 1 = 0$$
$$x = 1.4 \qquad x = 1$$

The answer is points where the line intersects the circle so we finally use the equation of the line $y = 2x - 3$ to work out y when x is 1.4, and 1.
FINAL ANSWER:

$$(1.4, -0.2) \ and \ (1, -1)$$

Phew !

8.2 Exercise

Question 1.
Solve the equation

$$2x^2 + 6x + 4 = 0$$

by first dividing throughout by 2.

Question 2.
Solve the equation

$$3x^2 + 15x + 18 = 0$$

by first dividing throughout by 3.

Question 3.
Solve the simultaneous equations

$$y = x - 4$$
$$x^2 + y^2 = 58$$

Question 4.
GCSE, June 2007, paper 3H, Q19
Solve the simultaneous equations

$$y = 3x - 1$$
$$x^2 + y^2 = 5$$

Question 5.

Solve the simultaneous equations

$$y = x - 7$$

$$x^2 + y^2 = 109$$

Question 6.

Solve the simultaneous equations

$$y = 2x - 3$$
$$x^2 + y^2 = 18$$

8.3 Answers

8.3.1 Solutions (8.2 Exercise)

Answer 1.

$$2x^2 + 6x + 4 = 0$$
$$x^2 + 3x + 2 = 0$$
$$(x + 2)(x + 1) = 0$$
$$EITHER \quad x + 2 = 0 \quad OR \quad x + 1 = 0$$
$$Solutions : x = -2, -1$$

Answer 2.

$$3x^2 + 15x + 18 = 0$$
$$x^2 + 5x + 6 = 0$$
$$(x + 3)(x + 2) = 0$$
$$EITHER \quad x + 3 = 0 \quad OR \quad x + 2 = 0$$
$$Solutions : x = -3, -2$$

Answer 3.

$$x^2 + (x - 4)^2 = 58$$
$$x^2 + x^2 - 8x + 16 = 58$$
$$2x^2 - 8x - 42 = 0$$
$$after\ dividing\ through\ by\ 2 \quad x^2 - 4x - 21 = 0$$
$$(x + 3)(x - 7) = 0$$
$$EITHER \quad x + 3 = 0 \quad OR \quad x - 7 = 0$$
$$Solutions : x = -3,\ 7$$
$$\therefore\ Points\ of\ intersection\ are\ (-3, -7)\ and\ (7,3)$$

Answer 4.
$5x^2 - 3x - 2 = 0$
(- 0.4, - 2.2) & (1, 2)

Answer 5.
$x^2 - 7x - 30 = 0$
(-3, -10) & (10, 3)

Answer 6.
$5x^2 - 12x - 9 = 0$
(-0.6, -4.2) & (3, 3)