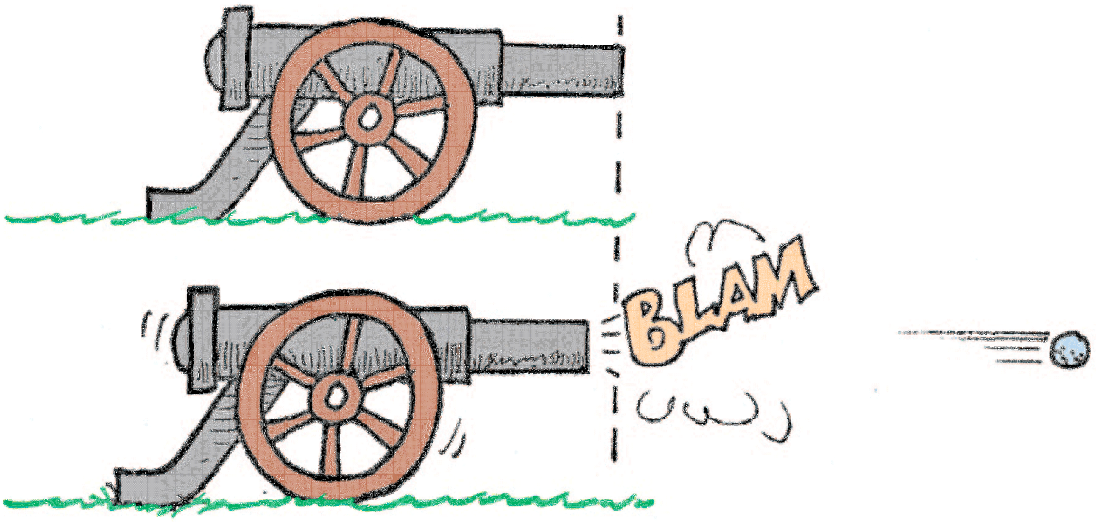
M O M E N T U M

&

I M P U L S E



Chapter 1

Momentum & Impulse : Mechanics 1

1.1 When Particles Collide

In this topic we consider what happens when particles collide at velocities well below the speed of light. The key idea is to look at the total momentum of the system before and, separately, after a collision has taken place. Although, typically, energy is lost† in such collisions, the total momentum before and after the collision is the same.

We say that "momentum is conserved"

1.2 The Theory

Definition of Momentum

$$momentum = mass \times velocity$$

The Conservation of Momentum Principal

$$total\ momentum\ before\ collision = total\ momentum\ after\ collision$$

Kinetic Energy (the energy of movement)

$$KE = \frac{1}{2} m v^2$$

Impulse

Although the total momentum of the system just before and just after a collision is conserved, for an individual particle it changes.
The change, called the Impulse, can be thought of as a 'tug', 'kick' or 'jerk'.

$$Impulse = change\ in\ momentum$$

$$I = mv - mu$$

Where m is the mass of the individual particle (which will not change),
u is the velocity of that particle before the collision
and v is the velocity of that particle after the collision

† You could argue that energy is conserved but in a collision energy gets converted to sound, light, static charge and also does work in deforming the objects being modelled by the particles in the collision.
It is impractical to a precise keep track of it all - and so the difference between the Kinetic Energy before and after the collision is referred to as the energy lost. This is a somewhat misleading expression as a principal of physics is that energy can neither be created nor destroyed, although it can transform into mass. ($E = m c^2$)

1.3 Example

Balls of mass 6 kg and 4 kg are moving directly towards each other with speeds of 7 ms^{-1} and 2 ms^{-1} respectively.
They collide.
The direction of the 6 kg mass is unchanged by the collision, it's speed then being reduced to 4 ms^{-1}

(i) Find the speed of the 4 kg mass after the collision.

(ii) Determine the magnitude of the Impulse on the 4 kg mass.

(iii) How much Kinetic Energy (if any) has been lost in the collision ?

1.4 Exercise

Question 1

A small smooth sphere *A* of mass 2.5 kg lies at rest on a smooth horizontal table. A second small smooth sphere *B* of mass 1.5 kg is moving with speed 4 ms^{-1} and collides directly with *A*. The two spheres *coalesce* - in other words, after impact they move as a single body. Find their speed after impact.

Question 2

M1 Exam question, 12th January 2005, Q1

A particle *P* of mass 1.5 kg is moving along a straight horizontal line with speed 3 ms^{-1}. Another particle *Q* of mass 2.5 kg is moving, in the opposite direction, along the same straight line with speed 4 ms^{-1}.
The particles collide.
Immediately after the collision the direction of motion of *P* is reversed and its speed is 2.5 ms^{-1}.

(a) Calculate the speed of *Q* immediately after the impact.

[3 marks]

(b) State whether or not the direction of motion of *Q* is changed by the collision.

[1 mark]

(c) Calculate the magnitude of the impulse exerted by *Q* on *P*, giving the units of your answer.

[3 marks]

Question 3

M1 Exam question, 3rd June 2015, Q1

Particle P of mass m and particle Q of mass km are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of P is $5u$ and the speed of Q is u. Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

Find

(a) the value of k

[3 marks]

(b) the magnitude of the impulse exerted on P by Q in the collision

[3 marks]

Question 4

M1 Exam question, 5th November 2002, Q6

A railway truck *P* of mass 1500 kg is moving on a straight horizontal track. The truck *P* collides with a truck *Q* of 2500 kg at a point *A*.
Immediately before the collision, *P* and *Q* are moving in the same direction with speeds of 10 ms^{-1} and 5 ms^{-1} respectively.
Immediately after the collision, the direction of motion of *P* is unchanged and its speed is 4 ms^{-1}.

By modelling the trucks as particles,

(a) show that the speed of *Q* immediately after the collision is 8.6 ms^{-1}.

[3 marks]

After the collision at *A*, the truck *P* is acted upon by a constant braking force of magnitude 500 N. The truck *P* comes to rest at the point *B*.

(b) Find the distance *AB*.

[3 marks]

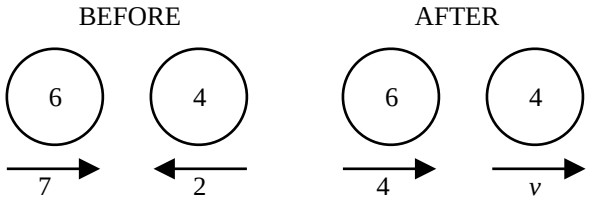
After the collision *Q* continues to move with constant speed 8.6 ms^{-1}.

(c) Find the distance between *P* and *Q* at the instant when *P* comes to rest.

[5 marks]

1.5 Answers

1.5.1 Solution to 1.3 Example



(i) CoM

$$6 \times 7 + 4 \times (-2) = 6 \times 4 + 4v$$

$$v = 2.5\, m/s$$

(ii)

$$I = mv - mu$$

$$= 4 \times 2.5 - 4 \times (-2)$$

$$= 18\, kg\, ms^{-1}$$

(iii)

$$KE_{before} = 0.5 \times 6 \times 7^2 + 0.5 \times 4 \times (-2)^2$$

$$= 155\, J$$

$$KE_{after} = 0.5 \times 6 \times 4^2 + 0.5 \times 4 \times 2.5^2$$

$$= 60.5\, J$$

$$\therefore \quad KE_{LOST} = 94.5\, J$$

1.5.2 Solution to 1.4 Exercise

Answer 1

1.5 m/s

Answer 2

(a) 0.7 m/s

(b) unchanged

(c) 8.25 kg m s^{-1}

Answer 3

(a) $k = 5$

(b) 7.5 *mu*

Answer 4

(a) CoM $1500 \times 10 + 2500 \times 5 = 1500 \times 4 + 2500 \times v$ => $v = 8.6$ ms^{-1}

(b) $F = ma$ => $1500\, a = -500$ => $a = -\frac{1}{3}$ ms^{-2}

$v^2 = u^2 + 2as$ => $s = 24$ m

(c) $v = u + at$ => $t = 12$ s

$s = ut$ => $s = 103.2$ => Distance apart = 79.2 m

Chapter 2

Momentum & Impulse : Mechanics 1

2.1 A Second Formula For Impulse

Previously, Impulse was described as a *kick* or a *jerk* and we could measure its effect upon an object by measuring the change of momentum of the object.

$$I = mv - mu$$

Intuitively, to stop an oil tanker using a tug, (i.e. To take away its momentum) a tug needs to apply a stopping *force* over a sustained period of *time*. The tug is applying a slow motion, gentle, *kick* to the oil tanker. (So as not to damage it) This suggests that there is another formula for Impulse that will involve force and time, and this is indeed the case.

$$\begin{aligned} I &= mv - mu \\ &= m(v - u) \qquad But \qquad v = u + at \\ &\qquad\qquad\qquad so \quad v - u = at \\ &= m\,a\,t \\ &\qquad\qquad\qquad But \qquad F = ma \\ &= F\,t \end{aligned}$$

Thus we have a second formula for Impulse;

$$Impulse = Force \times Time$$

Notice that the units of Impulse can be either kg ms^{-1} or Ns

2.2 Example N° 1

A toy train is pushed along a straight track by a horizontal force of 3 N for 4 s. Find the gain in momentum of the train.

2.3 Example N° 2

M1 Exam question, 12th November 2007, Q4

A particle P of mass 0.3 kg is moving with speed u ms^{-1} in a straight line on a smooth horizontal table. The particle P collides directly with a particle Q of mass 0.6 kg, which is at rest on the table. Immediately after the particles collide, P has speed 2 ms^{-1} and Q has speed 5 ms^{-1}. The direction of motion of P is reversed by the collision.

Find

(a) the value of u

[4 marks]

(b) the magnitude of the impulse exerted by P on Q

[2 marks]

Immediately after the collision, a constant force of magnitude R newtons is applied to Q in the direction directly opposite to the direction of motion of Q. As a result Q is brought to rest in 1.5 s

(c) Find the value of R

[4 marks]

2.4 Exercise

Question 1

A particle of mass 12 kg is initially moving with velocity 3 ms^{-1} on a smooth horizontal surface. A 4 N horizontal force acts on the particle for 6 seconds. Find the final speed of the particle by using the fact that;

$$Ft = mv - mu$$

Question 2

A canon of mass 800 kg discharges a canonball of mass 20 kg.
The canon recoils against a constant force of 4 kN which brings it to rest in 1.25 s.
Find the speed of the shot.

Question 3

A canonball of mass 15 kg is fired with speed 560 ms^{-1}.
Given that the shell is in the barrel of the canon for 0.05 s, calculate the average force, in kN, exerted on the shell by the explosive charge.

Question 4

A concrete block of mass 2 kg falls from rest from a vertical height of 10 m above horizontal ground.

(a) Calculate the speed immediately before it hits the ground

The ground is soft and, after the concrete block reaches the ground, it sinks vertically downwards into the ground before coming to rest. The ground is assumed to exert a constant resistive force of magnitude 5600 N on the block.

(b) Find the vertical distance that the concrete block sinks into the ground before coming to rest. Give your answer in cm.

Question 5

M1 Exam question, 16th May 2012, Q5

A particle P is projected vertically upwards from a point A with speed u ms^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 ms^{-1}.

(a) Show that $u = 21$

[3 marks]

At time t seconds after projection, P is 19 m above A.

(b) Find the possible values of t.

[5 marks]

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P

(c) Find the vertical distance that P sinks into the ground before coming to rest.

[4 marks]

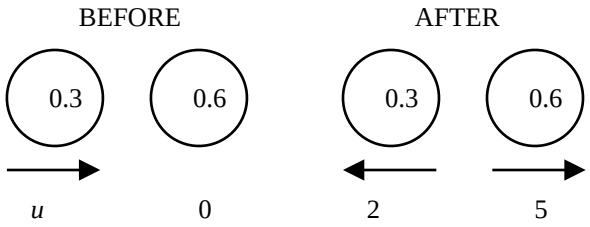
2.5 Answers

2.5.1 Solution to 2.2 Example N° 1

"Gain in momentum" is the same as the "change in momentum" of the toy train so the question is simply asking that the Impulse be calculated.

$$\begin{aligned} I &= Ft \\ &= 3 \times 4 \\ &= 12\,N \end{aligned}$$

2.5.2 Solution to 2.3 Example N° 2



(a) CoM

$$0.3 \times u + 0.6 \times 0 = 0.3 \times (-2) + 0.6 \times 5$$

$$0.3u = -0.6 + 3$$

$$u = 8\,m/s$$

(b) For Q

$$\begin{aligned} I &= mv - mu \\ &= 0.6 \times 5 - 0.6 \times 0 \\ &= 3\,Ns \end{aligned}$$

P has exerted a kick of magnitude 3 Ns upon Q

(c)

$$I = Ft$$

$$3 = F \times 1.5$$

$$F = 2\,N$$

Worth 4 marks as many exam candidates don't spot this easy way of doing the question !
(They use $v = u + at$ to get a, then use $F = ma$ to get F)

2.5.3 Solution to 2.4 Exercise

Answer 1

$$\begin{aligned} Ft &= mv - mu \\ 4 \times 6 &= 12 \times v - 12 \times 3 \\ 24 &= 12v - 36 \\ 12v &= 60 \\ v &= 5\,m/s \end{aligned}$$

Answer 2

$$\begin{aligned} Ft &= mv - mu \\ 4000 \times 1.25 &= 20v - 20 \times 0 \\ 5000 &= 20v \\ v &= 250\,m/s \end{aligned}$$

Answer 3

$$\begin{aligned} Ft &= mv - mu \\ F \times 0.05 &= 15 \times 560 - 15 \times 0 \\ 0.05F &= 8400 \\ F &= 168\,kN \end{aligned}$$

Answer 4

(a)

$$\begin{aligned} v^2 &= u^2 + 2as \\ v &= \pm\sqrt{0^2 + 2 \times (-9.8) \times (-10)} \\ v &= -14\,m/s \qquad \therefore \textit{speed is } 14\,m/s \end{aligned}$$

(b)

$$\begin{aligned} Ft &= mv - mu \\ (5600 + 2 \times (-9.8)\,t &= 2 \times 0 - 2 \times (-14) \\ t &= 0.00502\,s \end{aligned}$$

$$\begin{aligned} s &= \left(\frac{u + v}{2}\right) \times t \\ &= 7 \times 0.00502 \\ &= 0.035\,m \\ &\textit{i.e. } 3.5\,cm \end{aligned}$$

Answer 5

(a)

$$v^2 = u^2 + 2as$$
$$(-28)^2 = u^2 + 2 \times (-9.8) \times (-17.5)$$
$$784 = u^2 + 343$$
$$u^2 = 441$$
$$u = 21\, m/s \quad \textit{as expected}$$

(b)

$$s = ut + \frac{1}{2} a t^2$$
$$19 = 21t + 0.5 \times (-9.8)\, t^2$$
$$19 = 21t - 4.9 t^2$$
$$4.9 t^2 - 21t + 19 = 0$$
$$t = \frac{21 \pm \sqrt{(-21)^2 - 4 \times 4.9 \times 19}}{2 \times 4.9}$$
$$t = 2.99\, s,\ 1.30\, s$$

Note that *A* is the height zero level in these calculations

(c)

$$Ft = mv - mu$$
$$(5000 + 4 \times (-9.8))\, t = 4 \times 0 - 4 \times (-28)$$
$$t = 0.02258\, s$$

$$s = \left(\frac{u + v}{2}\right) \times t$$
$$= 14 \times 0.02258$$
$$= 0.316\, m$$

Missing out the $4 \times (-9.8)$ results in an answer of 0.3136 m

Alternatively the acceleration can be found as -1240.2 m/s^2 and then $v^2 = u^2 + 2as$ used to get the same answer.

Chapter 3

Momentum & Impulse : Mechanics 1

3.1 Examination Questions

3.2 Example

M1 Exam question, 14th January 2002, Q1

A ball of mass 0.3 kg is moving vertically downwards with speed 8 ms^{-1} when it hits the floor which is smooth and horizontal.
It rebounds vertically from the floor with speed 6 ms^{-1}.
Find the magnitude of the impulse exerted by the floor on the ball.

[3 marks]

3.3 Exercise

Question 1
M1 Exam question, 20th January 2012, Q1

A railway truck P, of mass m kg, is moving along a straight horizontal track with speed 15 ms^{-1}. Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is 3 ms^{-1} and the speed of Q is 9 ms^{-1}.
The direction of motion of P is reversed by the collision.

Modelling the trucks as particles, find

(a) the magnitude of the impulse exerted by P on Q

[2 marks]

(b) the value of m

[3 marks]

Question 2

M1 Exam question, 14th January 2002, Q2 (edited)

A railway truck *A* of mass 1800 kg is moving along a straight horizontal track with speed 4 ms^{-1}. It collides directly with a stationary truck *B* of mass 1200 kg on the same track. In the collision, *A* and *B* are coupled and move off together.

(a) find the speed of the trucks immediately after the collision.

[3 marks]

After the collision, the trucks experience a constant resistive force of magnitude *rR* newtons. They come to rest 8 s after the collision

(b) Find *R* by using the impulse relationship

$$Ft = mv - mu$$

[3 marks]

Question 3

M1 Exam question, 18th May 2011, Q2

Particle *P* has mass 3 kg and particle *Q* has mass 2 kg. The particles are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision, *P* has speed 3 ms^{-1} and *Q* has speed 2 ms^{-1}. Immediately after the collision, both particles move in the same direction and the difference in their speeds is 1 ms^{-1}.

(a) Find the speed of each particle after the collision.

[5 marks]

(b) Find the magnitude of the impulse exerted on *P* by *Q*.

[3 marks]

Question 4

M1 Exam question, 24th May 2010, Q2

Particle P has mass m kg and particle Q has mass $3m$ kg. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision, P has speed $4u$ ms^{-1} and Q has speed ku ms^{-1}, where k is a constant. As a result of the collision the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k

[4 marks]

(b) Find, in terms of m and u, the magnitude of the impulse exerted on P by Q.

[3 marks]

Question 5

M1 Exam question, 18th January 2001, Q5

Two small balls *A* and *B* have masses 0.6 kg and 0.2 kg respectively. They are moving towards each other in opposite directions on a horizontal table when they collide directly. Immediately before the collision, the speed of *A* is 4.5 ms^{-1} and the speed of *B* is speed 3 ms^{-1}. Immediately after the collision, *A* and *B* move in the same direction and the speed of *B* is twice the speed of *A*.

By modelling the balls as particles, find

(a) the speed of *B* immediately after the collision.

[4 marks]

(b) the magnitude of the impulse exerted on *B* in the collision stating the units in which your answer is given.

[3 marks]

The table is rough. After the collision, B moves a distance of 2 m on the table before coming to rest

(c) Find the coefficient of friction between B and the table.

[6 marks]

3.4 Answers

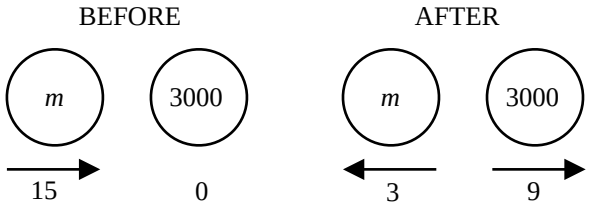
3.4.1 Solution to 3.2 Example

$$\begin{aligned} I &= mv - mu \\ &= 0.3 \times 6 - 0.3 \times (-8) \\ &= 1.8 + 2.4 \\ &= 4.2\ kg\ ms^{-1} \end{aligned}$$

Ball receives a *kick* upwards of 4.2 kg ms^{-1} from the floor.

3.4.2 Solutions to 3.3 Exercise

Answer 1



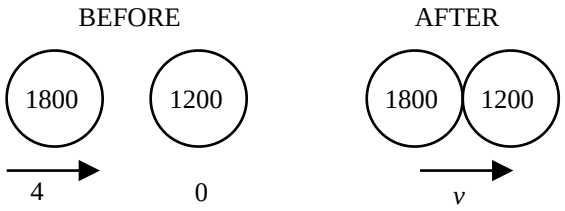
(a)

$$\begin{aligned} I &= mv - mu \\ &= 3000 \times 9 - 3000 \times 0 \\ &= 27\,000\ kg\ ms^{-1} \end{aligned}$$

(b) CoM

$$\begin{aligned} m \times 15 + 3000 \times 0 &= m \times (-3) + 3000 \times 9 \\ 15m &= -3m + 27\,000 \\ 18m &= 27\,000 \\ m &= 1500\ kg \end{aligned}$$

Answer 2



(a) CoM

$$1800 \times 4 + 1200 \times 0 = 1800 \times v + 1200 \times v$$

$$7200 = 3000v$$

$$v = 2.4\ ms^{-1}$$

(b)

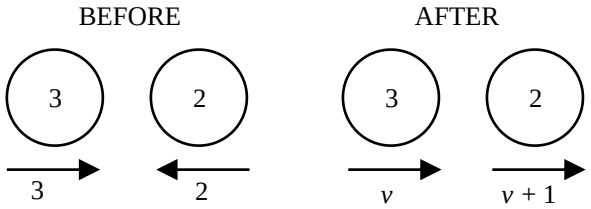
$$Ft = mv - mu$$

$$R \times 8 = 3000 \times 0 - 3000 \times 2.4$$

$$R = -900\ N$$

i.e. A constant resistive force of 900 *newtons*

Answer 3



(a) CoM

$$3 \times 3 + 2 \times (-2) = 3 \times v + 2 \times (v + 1)$$

$$5 = 3v + 2v + 2$$

$$5v = 3$$

$$v = 0.6\ ms^{-1}$$

Thus, after the collision P has speed 0.6 ms^{-1} and Q has speed 1.6 ms^{-1}

(b)

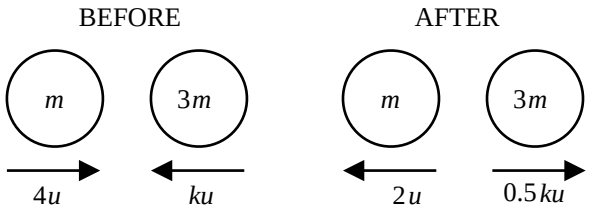
$$I = mv - mu$$

$$= 3 \times 0.6 - 3 \times 3$$

$$= -7.2\ kg\ ms^{-1}$$

Thus, P receives a *kick* of magnitude 7.2 kg ms^{-1} from Q

Answer 4



(a) CoM

$$m \times 4u + 3m \times (-ku) = m \times (-2u) + 3m \times 0.5ku$$

$$4mu - 3kmu = -2mu + 1.5kmu$$

$$6mu = 4.5kmu$$

$$6 = 4.5k$$

$$k = \frac{4}{3}$$

(b)

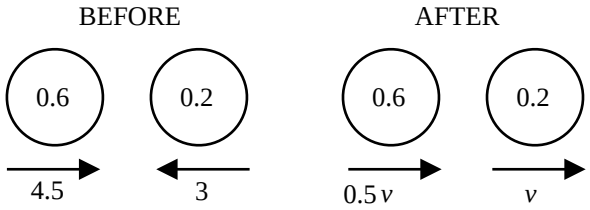
$$I = mv - mu$$

$$= m \times (-2u) - m \times 4u$$

$$= -6mu \; kg\; ms^{-1}$$

Thus, *P* receives a *kick* of magnitude $6mu$ kg ms^{-1} from *Q*

Answer 5



(a) CoM

$$0.6 \times 4.5 + 0.2 \times (-3) = 0.6 \times 0.5v + 0.2 \times v$$

$$2.7 - 0.6 = 0.3v + 0.2v$$

$$2.1 = 0.5v$$

$$v = 4.2\; ms^{-1}$$

(b)

$$I = mv - mu$$

$$= 0.2 \times 4.2 - 0.2 \times (-3)$$

$$= 0.84 + 0.6$$

$$= 1.44\; kg\; ms^{-1}$$

Thus, *B* receives a *kick* of magnitude 1.44 kg ms^{-1} from *A*

(c)

$$v^2 = u^2 + 2as$$
$$0^2 = 4.2^2 + 2a2$$
$$a = -4.41\ ms^{-2}$$

$$F = ma$$
$$= 0.2 \times (-4.41)$$
$$= -0.882\ N$$

Only force is that due to friction...

$$F_{FRICTION} = \mu R$$
$$0.882 = \mu mg$$
$$0.882 = \mu \times 0.2 \times 9.8$$
$$\mu = 0.45$$

Chapter 4

Momentum & Impulse : Mechanics 1

4.1 TEST

Question 1
M1 Exam question, 7th June 2001, Q1

Two small balls *A* and *B* have masses of 0.5 kg and 0.2 kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table when they collide directly. Immediately before the collision the speed of *A* is 3 ms^{-1} and the speed of *B* is 2 ms^{-1}.
The speed of *A* immediately after the collision is 1.5 ms^{-1}.
The direction of motion of *A* is unchanged as a result of the collision.

By modelling the balls as particles, find

(a) the speed of *B* immediately after the collision

[3 marks]

(b) the magnitude of the impulse exerted on *B* in the collision

[3 marks]

Question 2

M1 Exam question, 23rd May 2002, Q2

The masses of two particles *A* and *B* are 0.5 kg and *m* kg respectively. The particles are moving on a smooth horizontal table in opposite directions and collide directly. Immediately before the collision the speed of *A* is 5 ms^{-1} and the speed of *B* is 3 ms^{-1}.
In the collision, the magnitude of the impulse exerted by *B* on *A* is 3.6 Ns. As a result of the collision the direction of motion of *A* is reversed.

(a) Find the speed of *A* immediately after the collision

[3 marks]

The speed of *B* immediately after the collision is 1 ms^{-1}.

(b) Find the two possible values of *m*

[4 marks]

Question 3

M1 Exam question, 4th November 2003, Q2

A railway truck *S* of mass 2000 kg is travelling due east along a straight horizontal track with constant speed 12 ms^{-1}. The truck *S* collides with a truck *T* which is travelling due west along the same track as *S* with constant speed 6 ms^{-1}. The magnitude of the impulse of *T* on *S* is 28 800 Ns.

(a) Calculate the speed of *S* immediately after the collision

[3 marks]

(b) State the direction of motion of *S* immediately after the collision.

[1 mark]

Given that, immediately after the collision, the speed of *T* is 3.6 ms^{-1}, and that *T* ans *S* are moving in opposite directions

(c) calculate the mass of *T*

[4 marks]

Question 4

M1 Exam question, 12th January 2007, Q4

A particle P of mass 0.3 kg is moving with speed u ms^{-1} in a straight line on a smooth horizontal table. The particle P collides directly with a particle Q of mass 0.6 kg, which is at rest on the table.
Immediately after the particles collide, P has speed 2 ms^{-1} and Q has speed 5 ms^{-1}. The direction of motion of P is reversed by the collision.
Find

(a) the value of u

[4 marks]

(b) the magnitude of the impulse exerted by P on Q

[2 marks]

Immediately after the collision, a constant force of magnitude R newtons is applied to Q in the direction directly opposite to the direction of motion of Q. As a result Q is brought to rest in 1.5 s.

(c) Find the value of R

[4 marks]

Question 5

M1 Exam question, 2nd November 2004, Q4

A tent peg is driven into soft ground by a blow from a hammer.
The tent peg has mass 0.2 kg and the hammer has mass 3 kg.
The hammer strikes the peg vertically.
Immediately before the impact, the speed of the hammer is 16 ms^{-1}.
It is assumed that, immediately after the impact, the hammer and the peg move together vertically downwards.

(a) Find the common speed of the peg and the mammer immediately after impact.

[3 marks]

Until the peg and hammer come to rest, the resistance exerted by the ground is assumed to be constant and of magnitude *R* newtons. The hammer and peg are brought to rest 0.05 s after the impact.

(b) Find, to 3 significant figures, the value of *R*

[5 marks]

Question 6

M1 Exam question, 8th June 2016, Q3

A particle P of mass 0.4 kg is moving on rough horizontal ground when it hits a fixed vertical plane wall. Immediately before hitting the wall, P is moving with speed 4 ms^{-1} in a direction perpendicular to the wall. The particle rebounds from the wall and comes to rest at a distance of 5 m from the wall. The coefficient of friction between P and the ground is $\frac{1}{8}$

Find the magnitude of the impulse exerted on P by the wall.

[7 marks]