Transformation Geometry

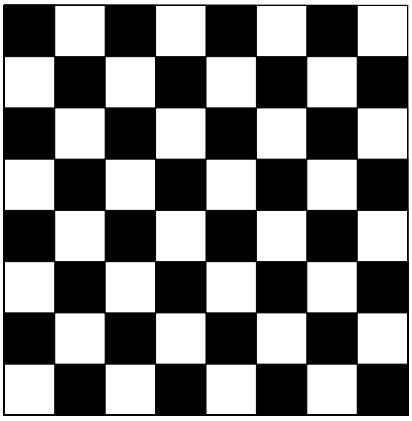
Chapter 1.

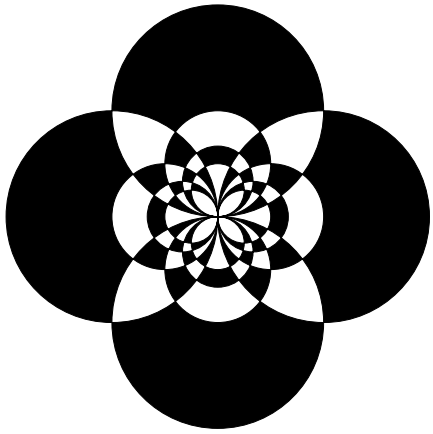
1.1 Introduction

In this presentation of **Transformation Geometry**, four TRANSFORMATIONS will be studied. In turn, each of the following will be considered:

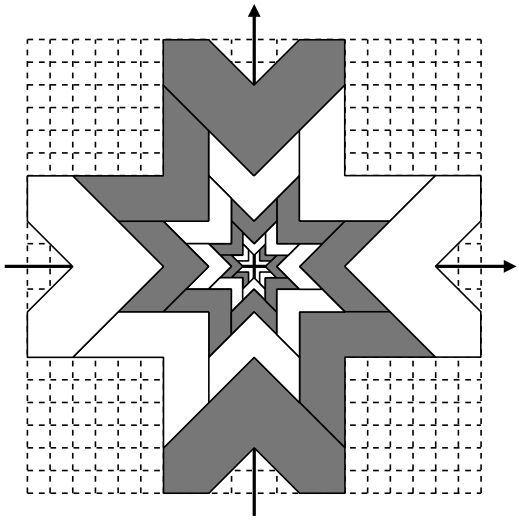
T1. Translation.
T2. Reflection.
T3. Rotation.
T4. Enlargement.

Be aware that other beautiful and interesting transformations exist. Given an unfamiliar transformation to investigate, a feel for what it does can be gained by applying the transformation to test shapes. For example, the transformation known as an **inversion** distorts a chessboard as shown:



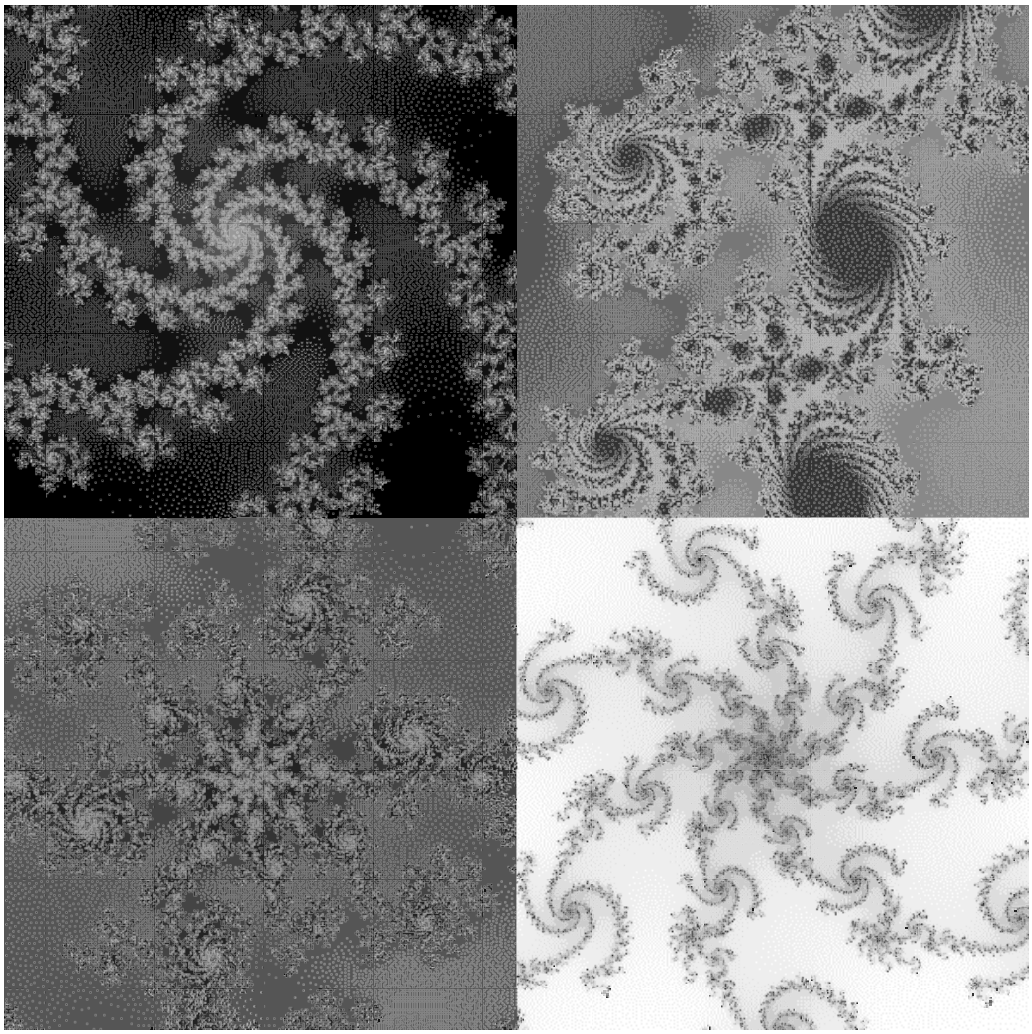


As a second example, here is the result of applying a transformation called a **spiral similarity** repeatedly to a V-shape.



The construction has been left unfinished to show the "work in progress".

With the development over the last thirty years of the computer, the true beauty of transformation geometry can be appreciated by all. Computers allow us to see in intricate detail an exciting and previously hidden mathematical world.



An internet search on the topic of "Transformations" will quickly find many fascinating images. The pictures above are pieces of the "Mandelbrot Set".

Notice that there are many spiral similarities in these picture.

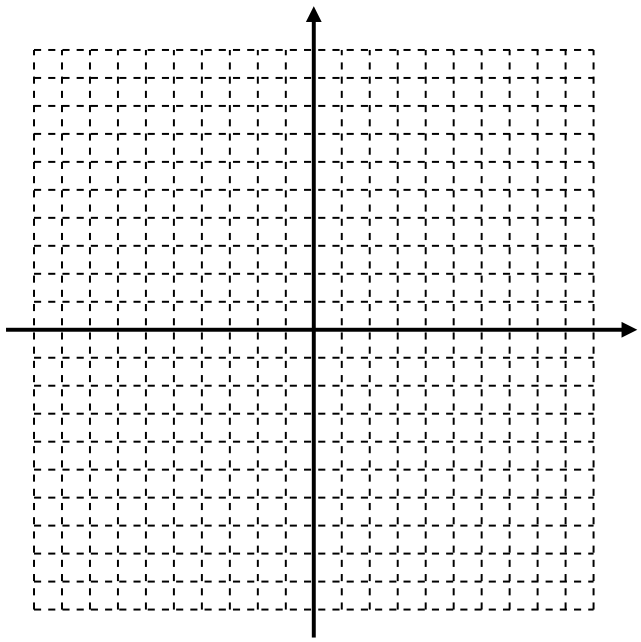
1.2 Exercise.
Search the internet to find some pictures of "transformations".
SearchWords : Fractals, Inversion, Spiral Similarities, Mandelbrot Set, Julia Set.

Chapter 2.

2.1 T1. Translation.

A ***translation*** is the first of the four ***transformations*** to be studied.

2.2 Example.



(i) Plot the KITE with vertices;
K (- 7, - 9), I (1, - 5), T (1, - 1), E (- 3, - 1).

(ii) The KITE has mirror symmetry.
On your diagram draw in the line of mirror symmetry.

(iii) What is the equation of this line of mirror symmetry ?

(iv) Translate the KITE by the vector; $\begin{bmatrix} 5 \\ 9 \end{bmatrix}$.

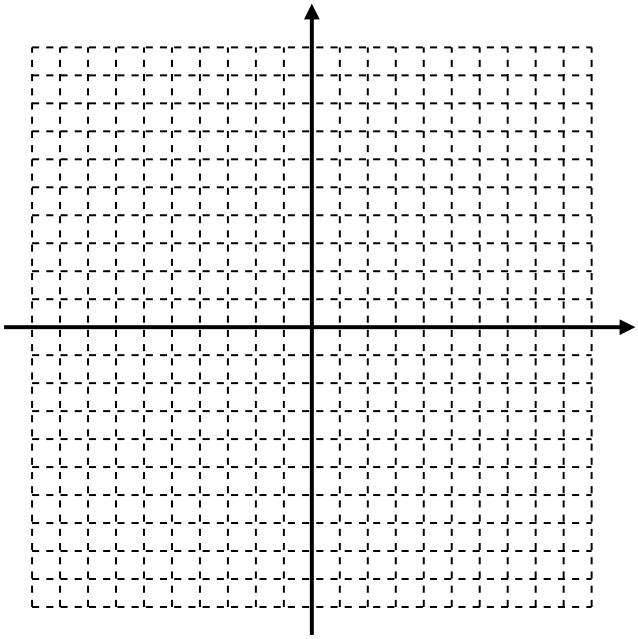
(v) What is the equation of the translated KITE's line of mirror symmetry ?

(vi) What can you say, geometrically, about the lines of symmetry of the original KITE and the translated KITE ?

(vii) Are the original KITE and the translated KITE congruent ?

2.3 Exercise.

Question 1.

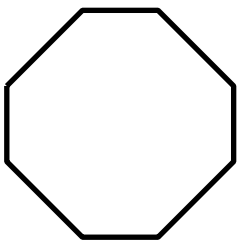


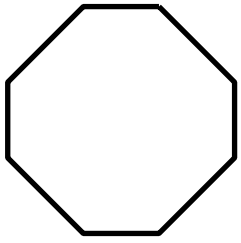
(i) Plot the PARALLELOGRAM with vertices;
G (- 9, - 6), R (- 5, - 6), A (- 1, 2), M (- 5, 2).

(ii) The PARALLELOGRAM does not have mirror symmetry.
It does have rotational symmetry of order 2.
About what point does it have rotational symmetry ?

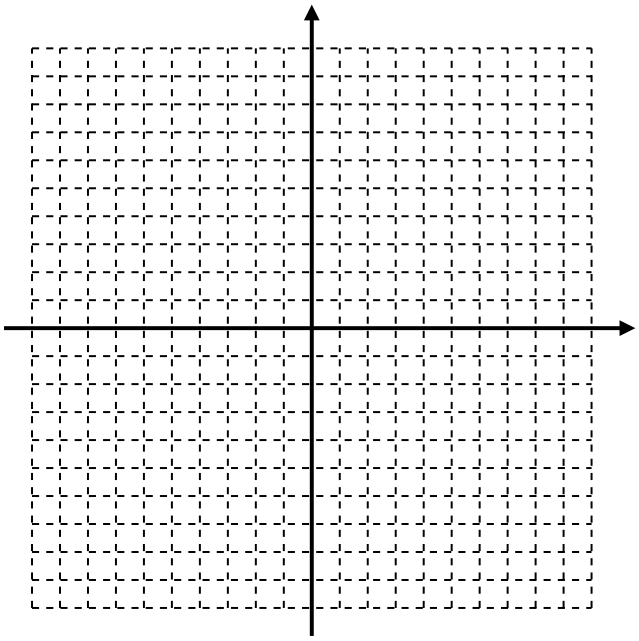
(iii) Translate the PARALLELOGRAM by the vector $\begin{bmatrix} 9 \\ 5 \end{bmatrix}$.

Question 2.
Use a cm ruler to help you write down a vector that translates the octagon on the left onto that on the right.





Question 3.



(i) Plot the RHOMBUS with vertices;
R (2, - 6), H (2, - 1), O (- 2, - 4), M (- 2, - 9).

(ii) The RHOMBUS has two lines of mirror symmetry.
On your diagram draw in the lines of mirror symmetry.

(iii) What are the equations of these lines of mirror symmetry ?

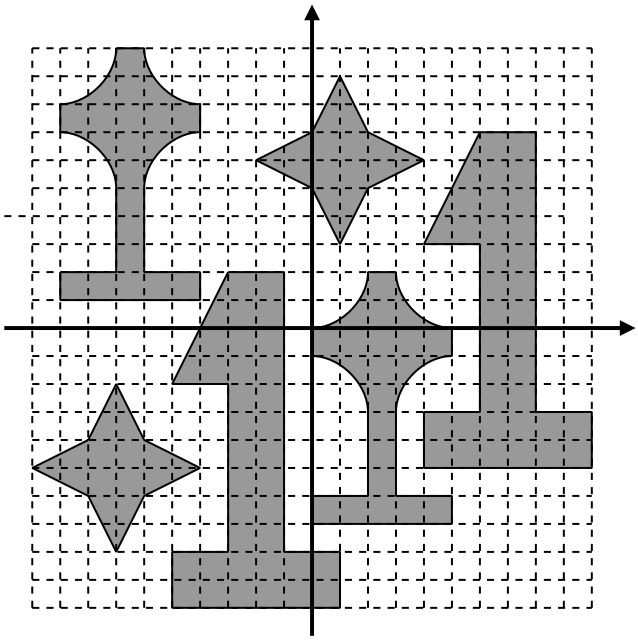
(iv) Translate the RHOMBUS by the vector; $\begin{bmatrix} -3 \\ 9 \end{bmatrix}$.

(v) What are the equations of the translated RHOMBUS's two lines of mirror symmetry ?

(vi) Does the RHOMBUS have rotational symmetry ?
If so, of what order ?

Question 4.
Draw a shape which has rotational symmetry of order 3 and no mirror symmetry.

Question 5.



(i) Write down the vector that translates the STAR, to the right and up, and onto its congruent twin.

(ii) Write down the vector that translates the ONE, to the left and down, and onto its congruent twin.

(iii) Write down the vector that translates the CELTIC CROSS to the right and down and onto its congruent twin.

Question 6.

(i) Translate the point (4, 7) by $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

(ii) Translate the point (24, 33) by $\begin{bmatrix} 21 \\ 37 \end{bmatrix}$.

Question 7.

(i) Translate the point (- 8, 3) by $\begin{bmatrix} 11 \\ -7 \end{bmatrix}$.

(ii) Translate the point (8.7, 6.3) by $\begin{bmatrix} 12.2 \\ -3.7 \end{bmatrix}$.

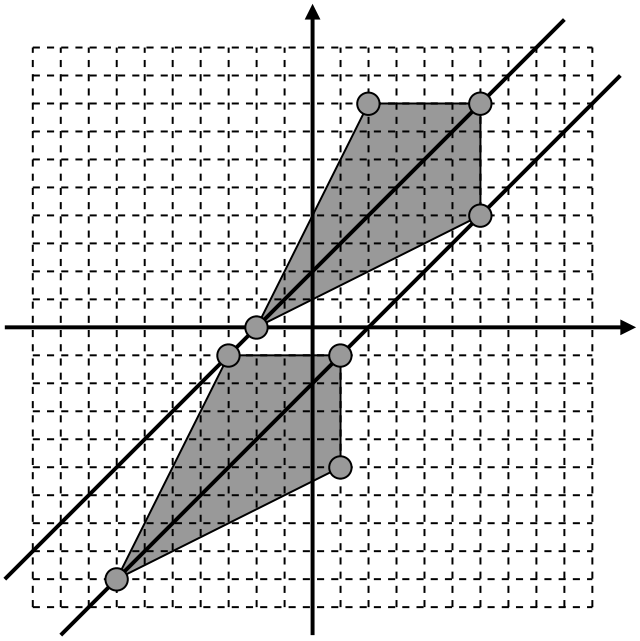
Question 8.

(i) What vector will translate the point (5, 1) onto the point (7, 11) ?

(ii) What vector will translate the point (4, 13) onto the point (-7, 8) ?

2.4 Answers.

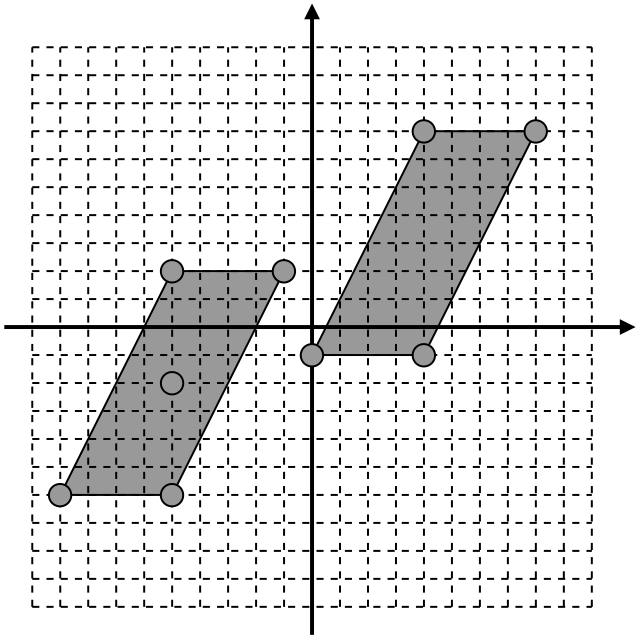
2.4.1 Solution (Example).



(iii) $y = x - 2$
(v) $y = x + 2$
(vi) parallel
(vii) yes

2.4.2 Solution (Exercise).

Answer 1.

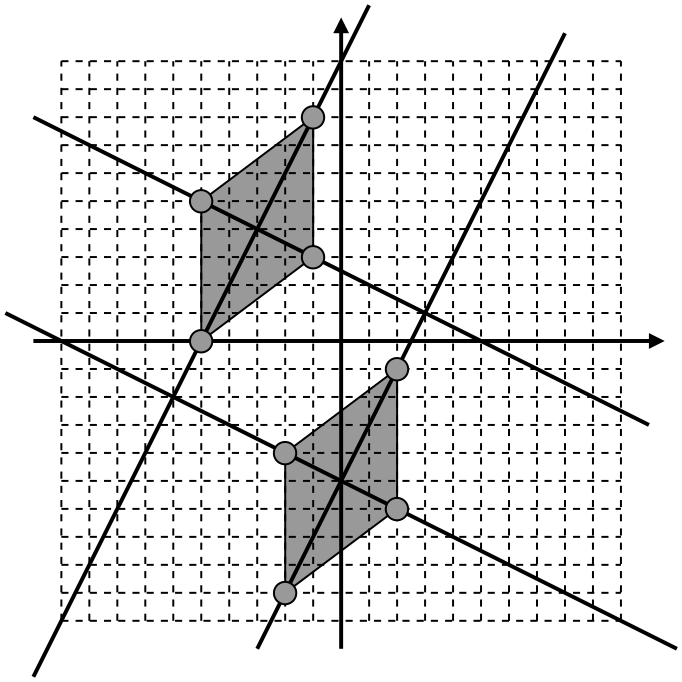


(ii) (- 5, - 2)

Answer 2.

Approximately $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ cm.

Answer 3.



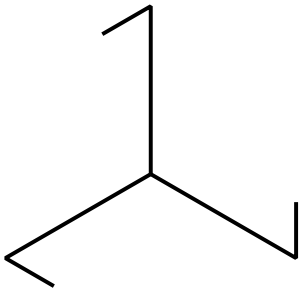
(iii) $y = 2x - 5$ and $y = -0.5x - 5$

(v) $y = 2x + 10$ and $y = -0.5x + 2.5$

(vi) It has rotational symmetry of order 2

Answer 4.

For example:



Answer 5.

(i) $\begin{bmatrix} 8 \\ 11 \end{bmatrix}$ **(ii)** $\begin{bmatrix} -9 \\ -5 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 9 \\ -8 \end{bmatrix}$

Answer 6.

(i) (9, 10)

(ii) (45, 70)

Answer 7.

(i) (3, - 4)

(ii) (20.9, 2.6)

Answer 8.

(i) $\begin{bmatrix} 2 \\ 10 \end{bmatrix}$

(ii) $\begin{bmatrix} -11 \\ -5 \end{bmatrix}$

Chapter 3.

3.1 T2. Reflection.

A ***reflection*** is the second of the four ***transformations*** to be studied.

In general, when a point or shape is transformed, the result is referred to as an *image*.

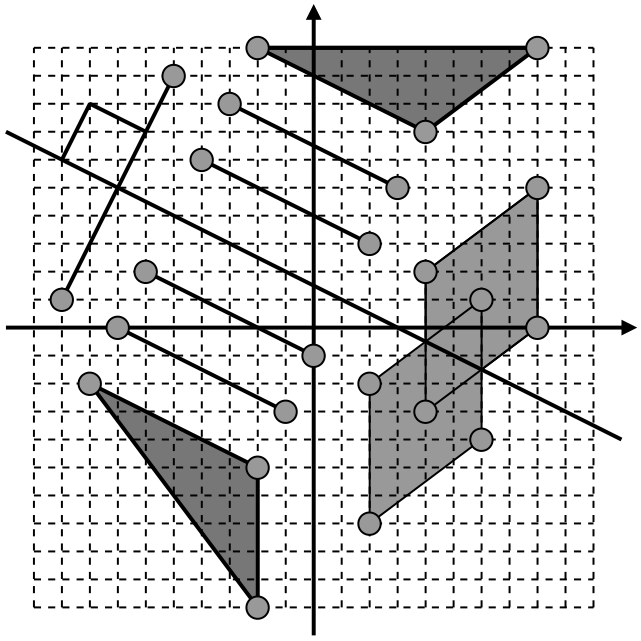
There are three key properties to keep in mind regarding reflections:

Ref 1. The perpendicular distance of any point from the mirror is equal to the perpendicular distance of the image from the mirror.

Ref 2. A line drawn from any point to its image passes through the mirror at 90°.

Ref 3. A line parallel to the mirror has a reflection that is also parallel to the mirror.

It is useful to have a *set square* to help answer questions involving reflection.



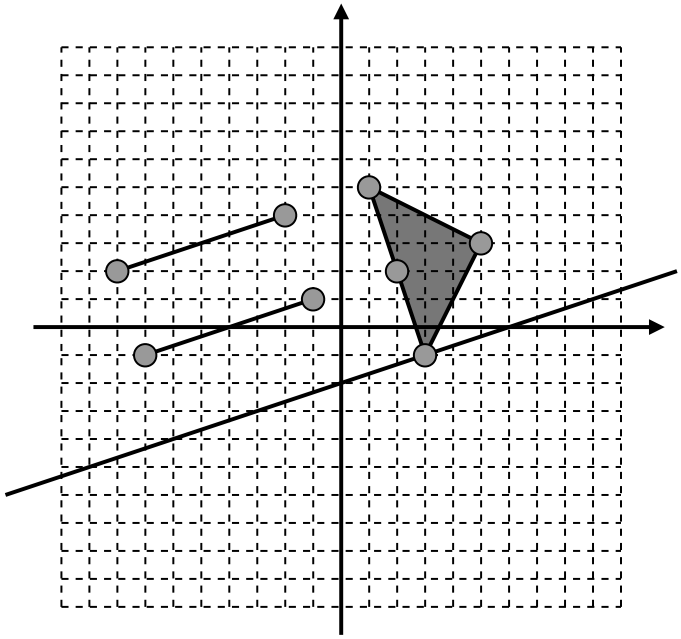
The mirror, being a straight line, has an equation of the form $y = m\,x + c$.

3.2 Example.

What is the equation of the mirror line, shown above ?

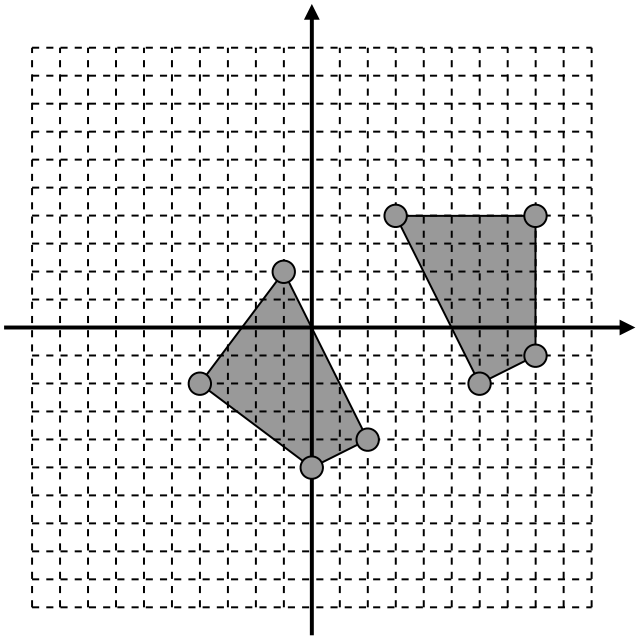
3.3 Exercise.

Question 1.



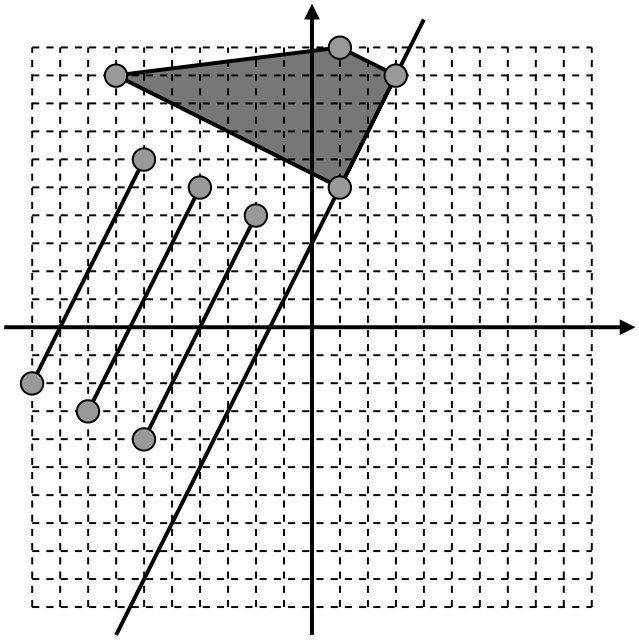
(i) Reflect each point, line and shape in the mirror line.
(ii) Write down the equation of the mirror line.

Question 2.



(i) Draw in the mirror.
(ii) Write down the equation of the mirror in the form $y = m\,x + c$.

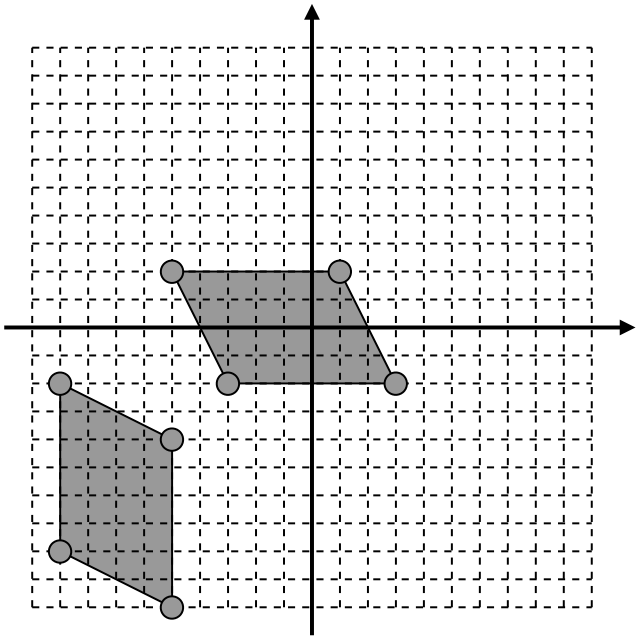
Question 3.



(i) Reflect each point, line and shape in the mirror line.

(ii) Write down the equation of the mirror line.

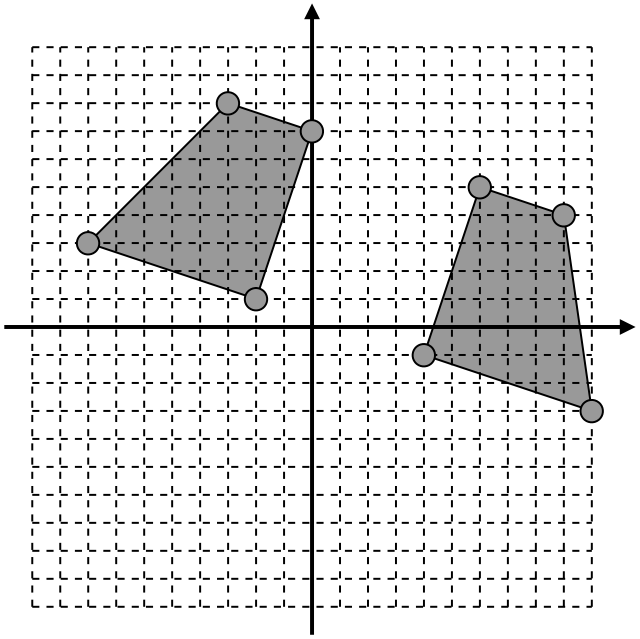
Question 4.



(i) Draw in the mirror.

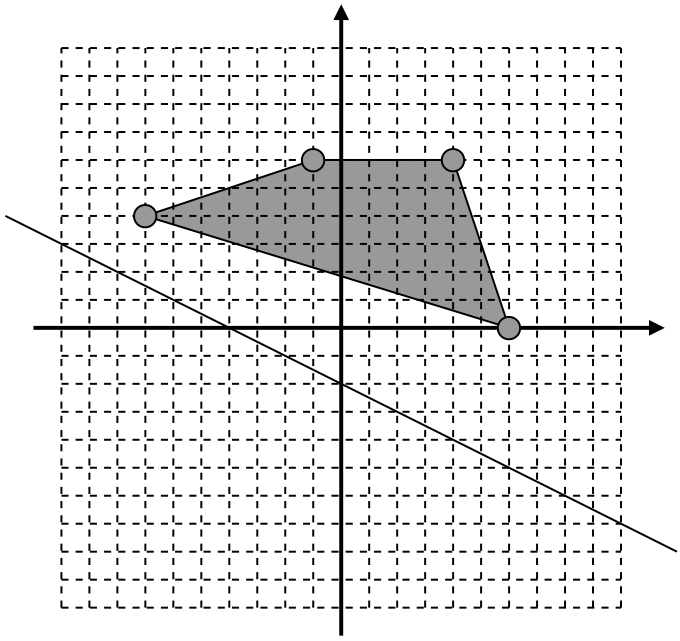
(ii) Write down the equation of the mirror in the form $y = m\,x + c$.

Question 5.



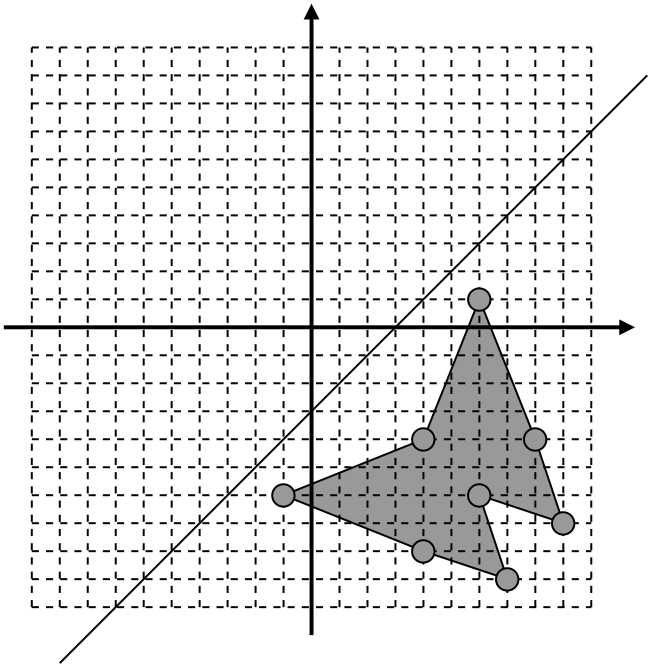
(i) Draw in the mirror.
(ii) Write down the equation of the mirror in the form $y = m\,x + c$.

Question 6.



(i) Reflect the shape in the mirror.
(ii) Write down the equation of the mirror in the form $y = m\,x + c$.

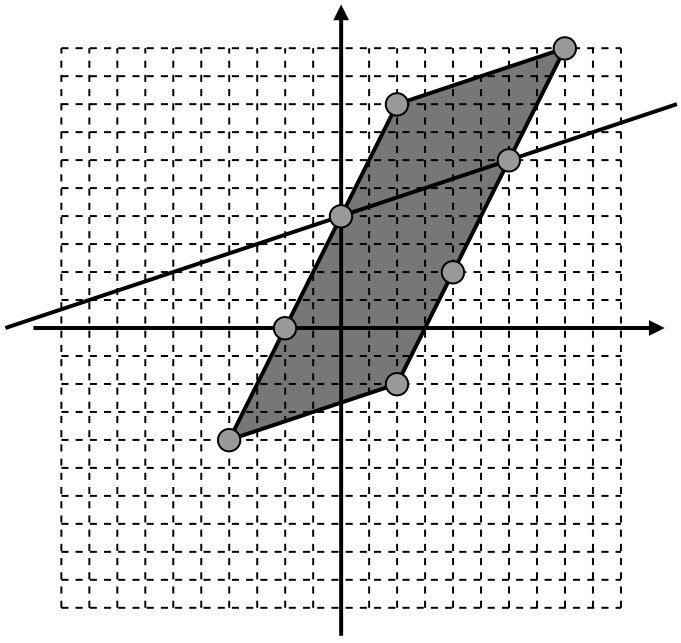
Question 7.



(i) Reflect the shape in the mirror.

(ii) Write down the equation of the mirror.

Question 8.



(i) Reflect the shape in the mirror.

(ii) Write down the equation of the mirror.

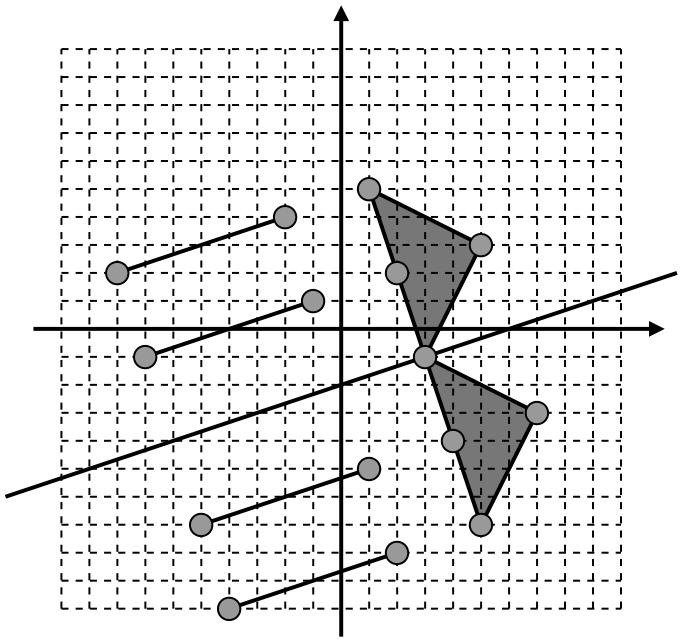
3.4 Answers.

3.4.1 Solution (Example).

$y = -0.5\,x + 1.5$

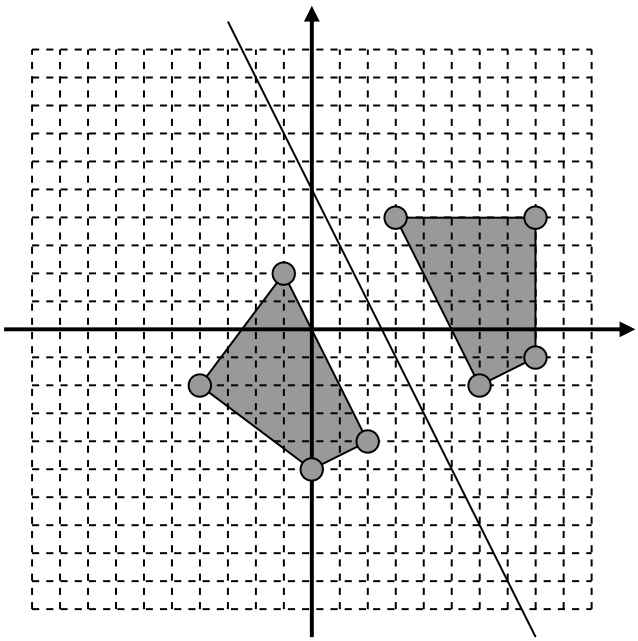
3.4.2 Solution (Exercise).

Answer 1.



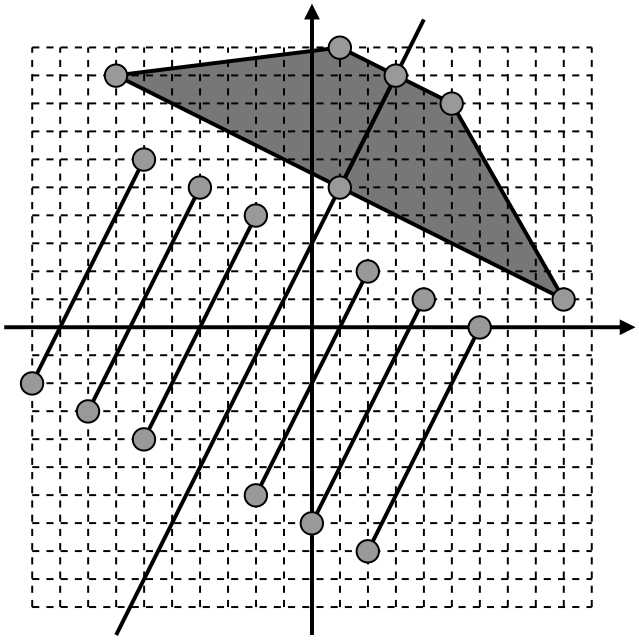
(ii) $y = \frac{1}{3}\,x - 2$

Answer 2.



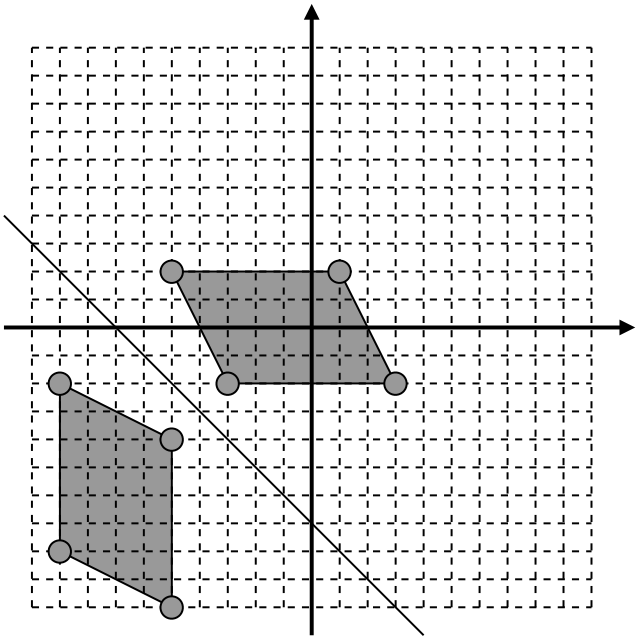
(ii) $y = -2\,x + 5$

Answer 3.



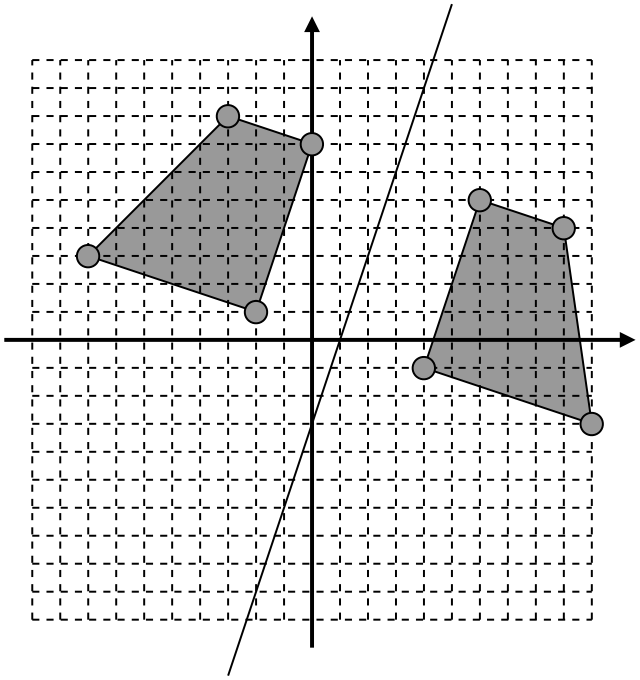
(ii) $y = 2x + 3$

Answer 4.



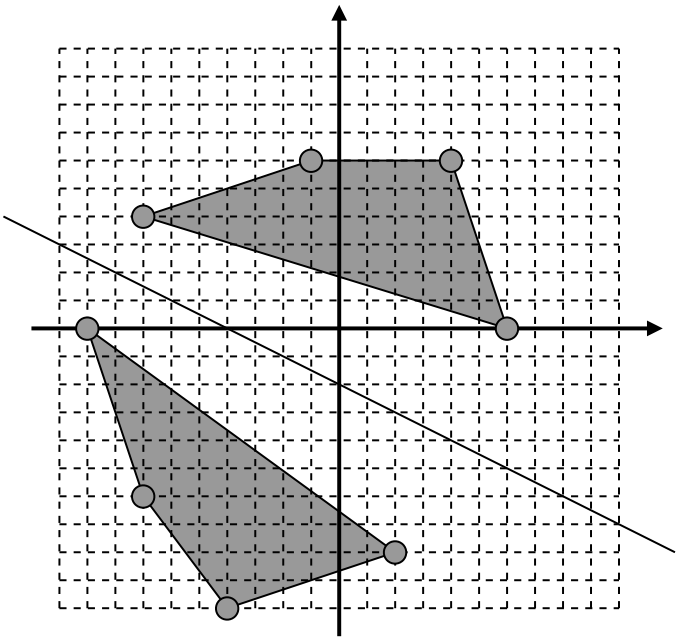
(ii) $y = -x - 7$

Answer 5.



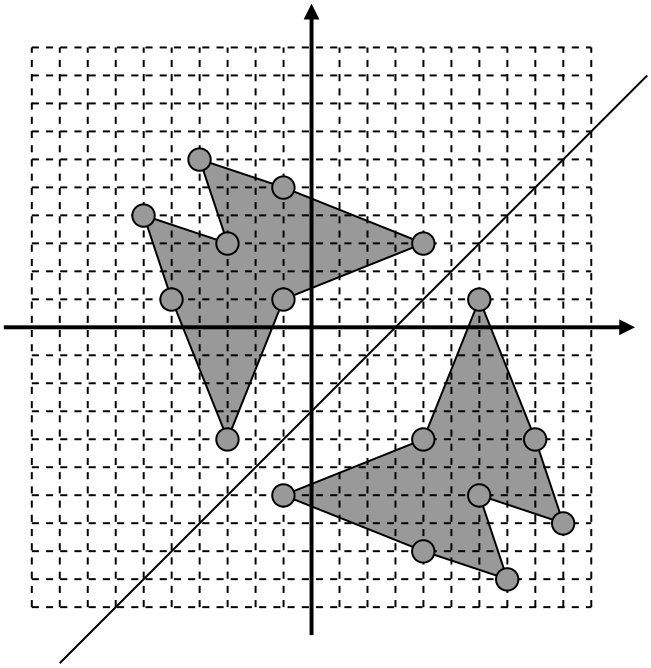
(ii) $y = 3x - 3$

Answer 6.



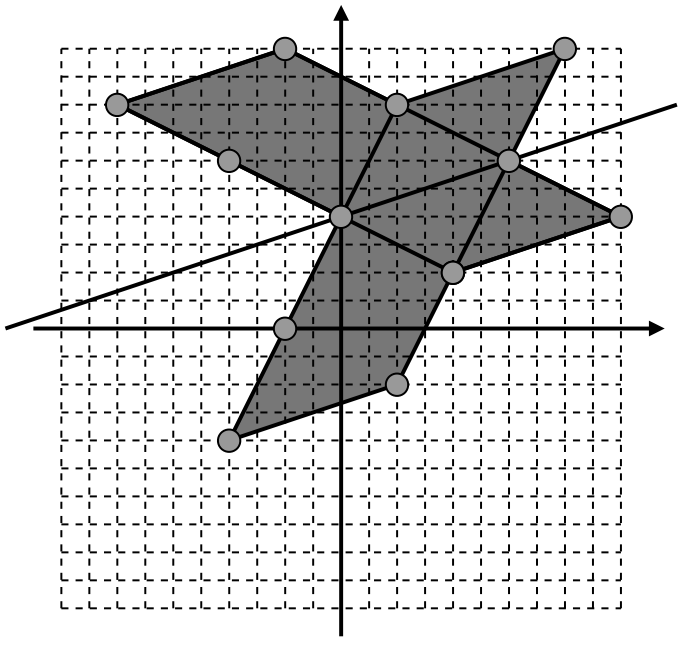
(ii) $y = -0.5x - 2$

Answer 7.



(ii) $y = x - 3$

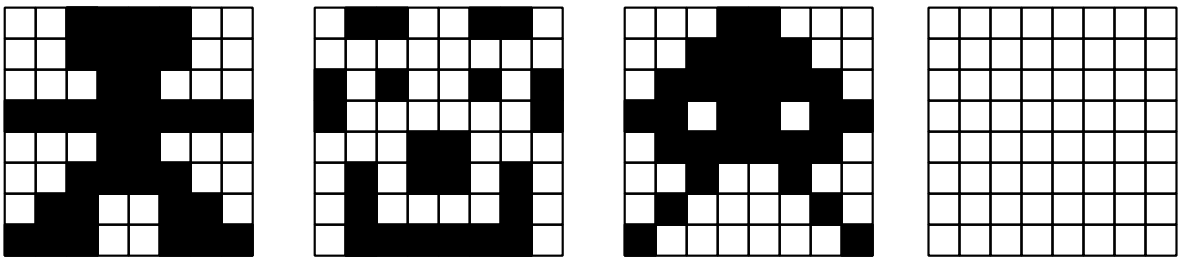
Answer 8.



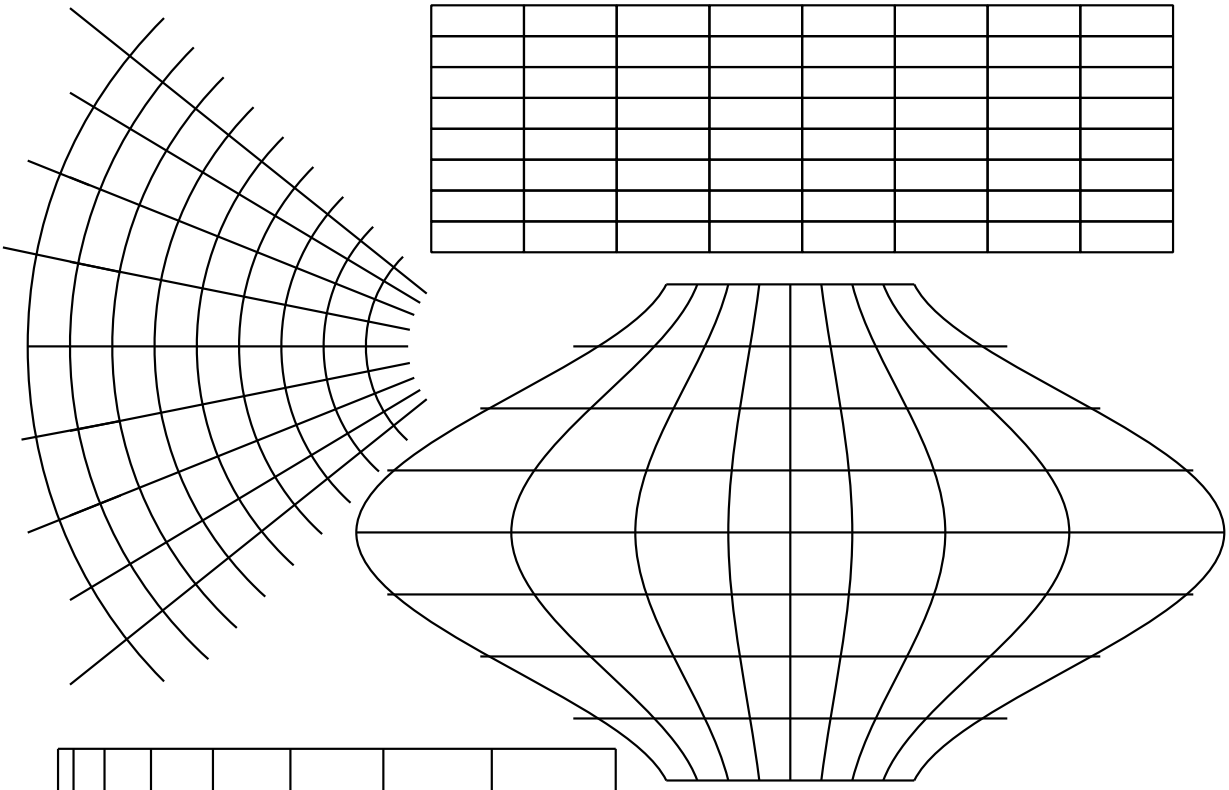
(ii) $y = \frac{1}{3}x + 4$

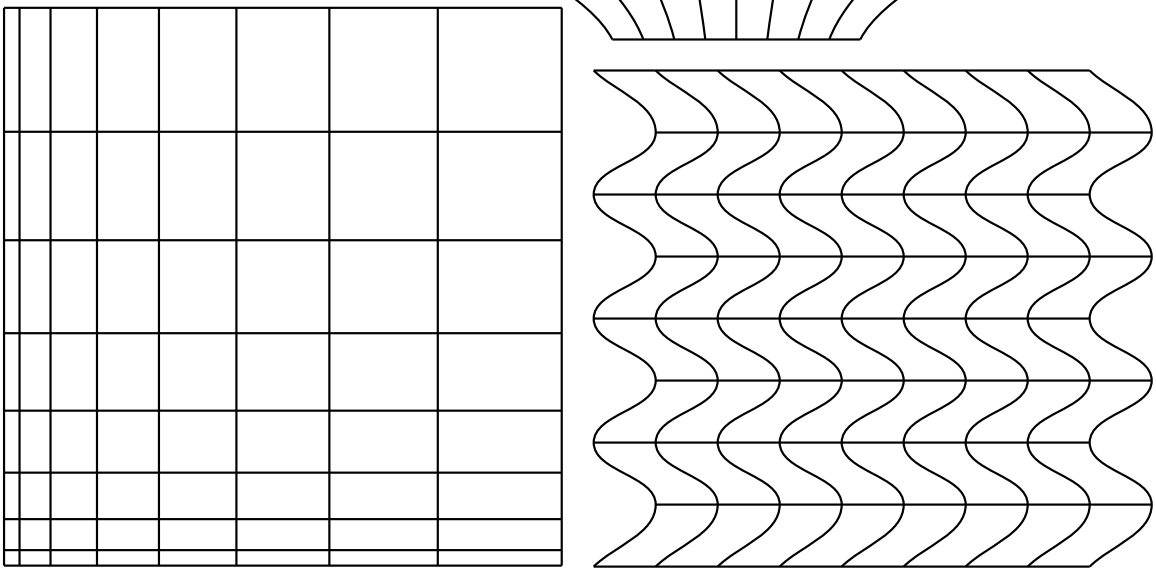
Chapter 4.

Weird and Wacky Transformations



Pick a design and transfer it onto the distortion grids





Chapter 5.

5.1 T3. Rotation.

A ***rotation*** is the third of the four ***transformations*** to be studied.

Three items of information describe a rotation fully:

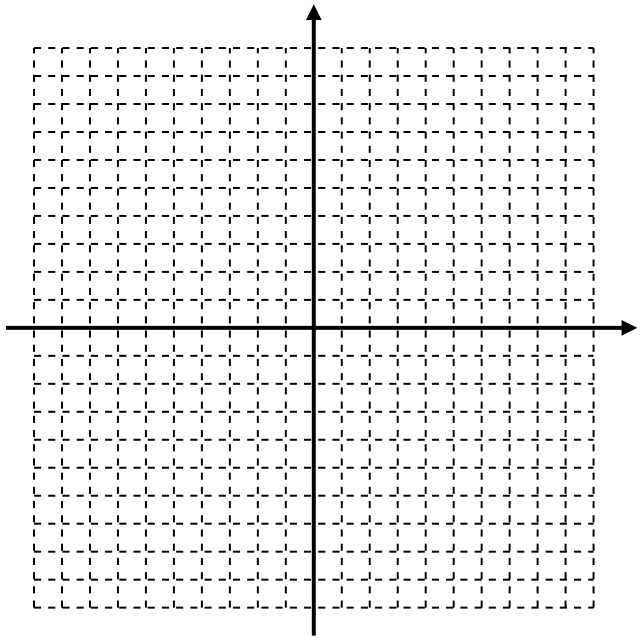
Ro1. The size of the **angle** of rotation.

Ro2. The **point** about which rotation takes place.

Ro3. The **direction** of the rotation: clockwise or anti-clockwise.

If the direction is omitted then it is assumed to be anti-clockwise.
In mathematics anti-clockwise rotation is considered to be positive.

5.2 Example.



(i) Plot the L-shape, *L*, with vertices;

A (0, - 4), *B* (0, - 10), *C* (6, - 10),
D (6, - 8), *E* (2, - 8), *F* (2, - 4).

Put the identifier *L1* inside the L-shape.

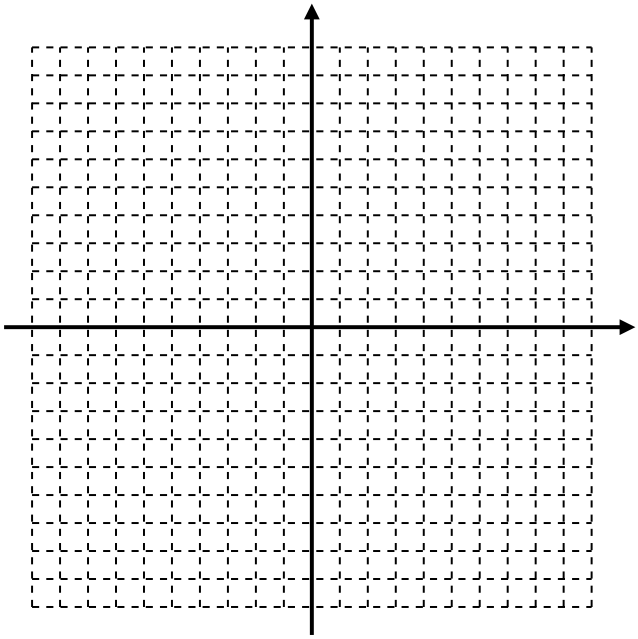
(ii) Rotate *L1* about the point (1, - 1) by 90°.
Put the identifier *L2* inside the rotated shape.

(iii) Rotate *L2* about the point (1, - 1) by 180°.
Put the identifier *L3* inside the rotated shape.

(iv) Finally, rotate *L3* about the point (1, - 1) by 270°.
Put the identifier *L4* inside the rotated shape.

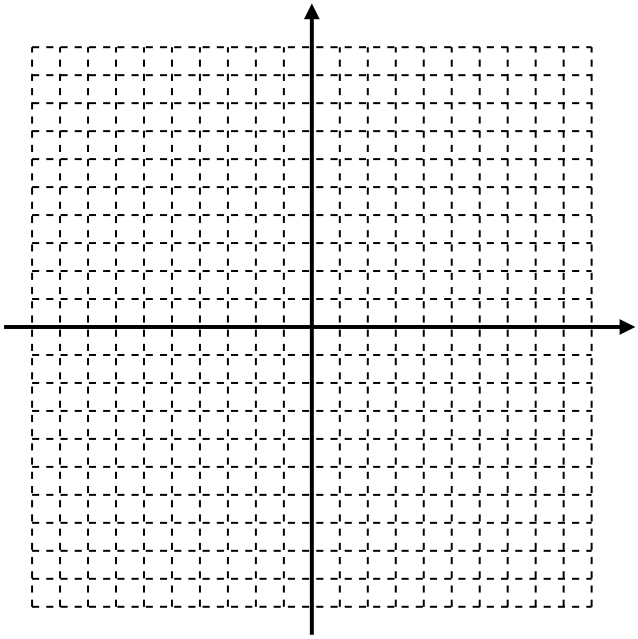
5.3 Exercise.

Question 1.



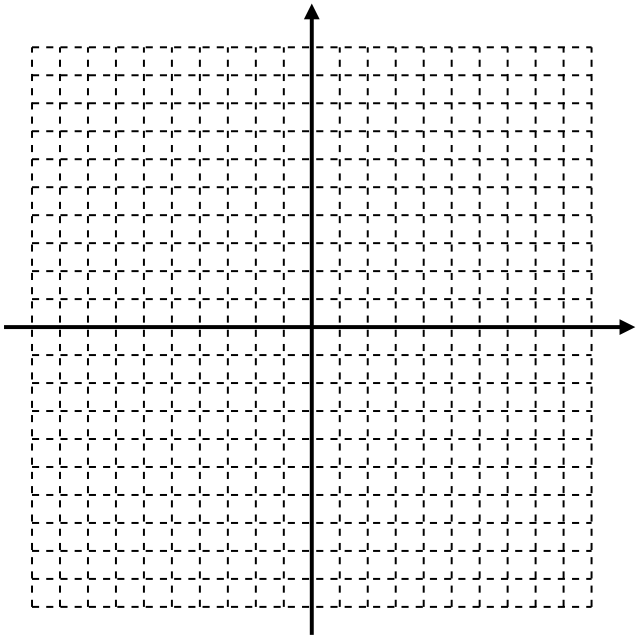
(i) Plot the triangle, *T*, with vertices; *A* (7, 4), *B* (3, 2), *C* (5, 0).
(ii) Put the identifier *T1* inside the triangle.
(iii) Rotate *T1* about the point (- 1, 2) by 90°.
(iv) Put the identifier *T2* inside the rotated shape.

Question 2.



(i) Plot the isosceles triangle, *I*; *J* (- 7, 2), *K* (- 1, 5), *L* (- 10, 8).
(ii) Put the identifier *I1* inside the isosceles triangle.
(iii) Rotate *I1* about the point (- 1, - 1) by - 90°.
(iv) Put the identifier *I2* inside the rotated shape.

Question 3.



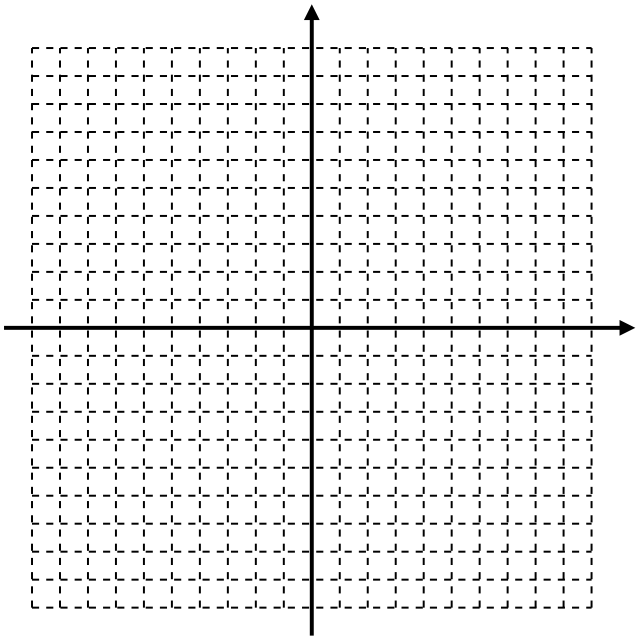
(i) Plot the rhombus, *R*, with vertices;
D (5, 3), *E* (- 3, 1), *F* (- 5, - 7), *G* (3, - 5).

(ii) Put the identifier *R1* inside the rhombus.

(iii) Rotate *R1* about the point (2, 0) by 180°.

(iv) Put the identifier *R2* inside the rotated shape.

Question 4.



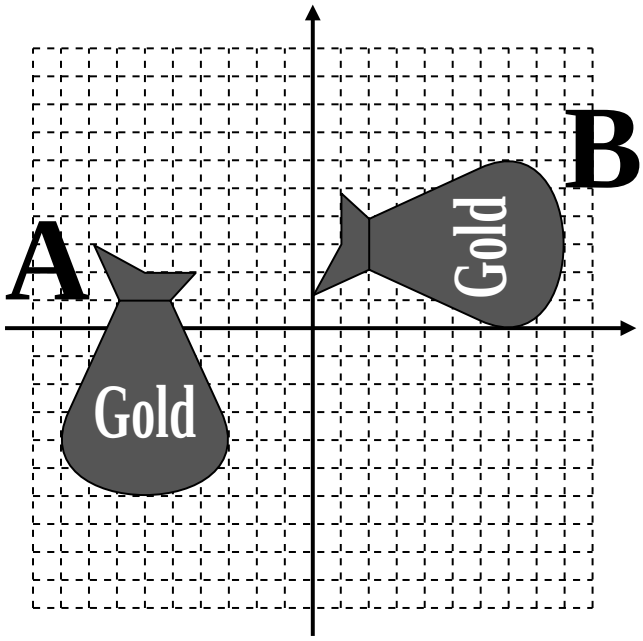
(i) Plot the kite, *K1*, with vertices;
P (- 9, - 1), *Q* (3, - 3), *R* (5, - 1), S (3, 1).

(ii) Put the identifier *K1* inside the kite.

(iii) Rotate *K1* about the point (1, 4) by 90°.

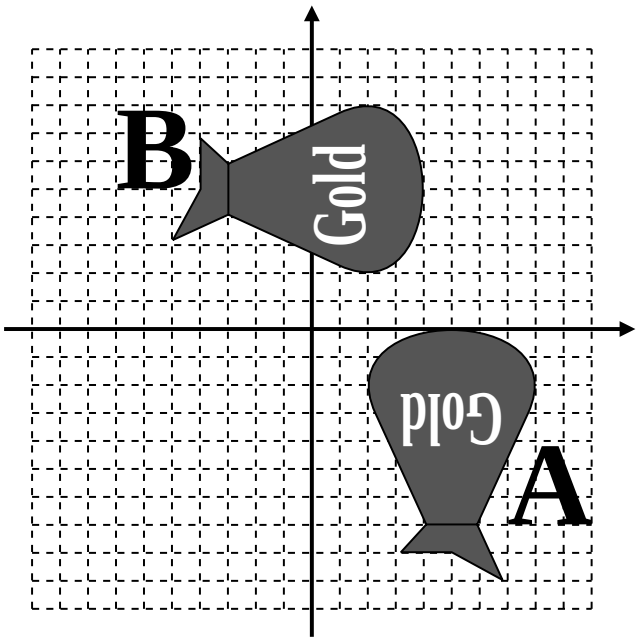
(iv) Put the identifier *K2* inside the rotated shape.

Question 5.



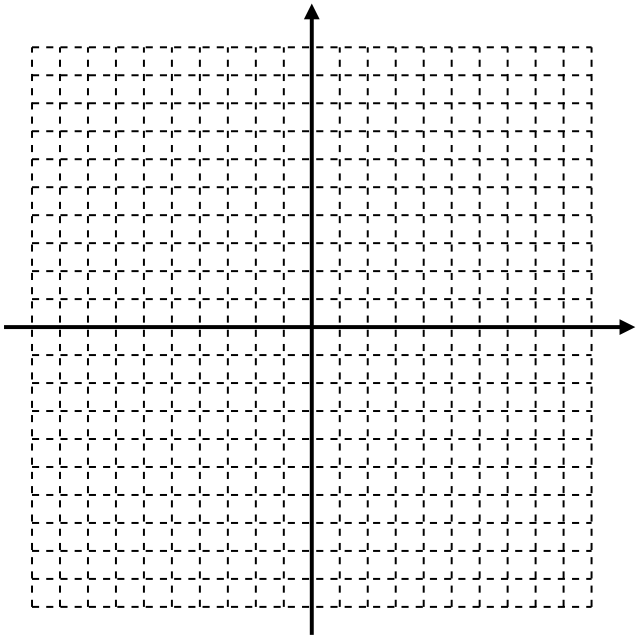
Specify fully the single transformation that maps bag of gold A onto bag of gold B.

Question 6.



Specify fully the single transformation that maps bag of gold A onto bag of gold B.

Question 7.



(i) Plot the Z-shape, *Z*, with vertices;

A (3, 3), *B* (3, 5), *C* (2, 5), *D* (2, 4),
E (- 4, 4), *F* (- 4, 0), *G* (- 2, 0), *H* (- 2, 3)

(ii) Put the identifier *Z1* inside the Z-shape.

(iii) Rotate *Z1* about the point (3, 1) by 90°.

(iv) Put the identifier *Z2* inside the rotated shape.

(v) Rotate *Z2* about the point (3, 1) by 180°.

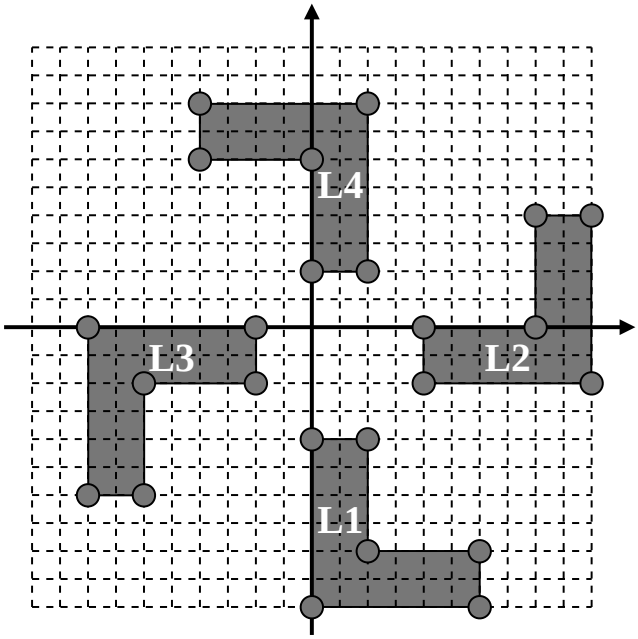
(vi) Put the identifier *Z3* inside the rotated shape.

(vii) Finally, rotate *Z3* about the point (3, 1) by 270°.

(viii) Put the identifier *Z4* inside the rotated shape.

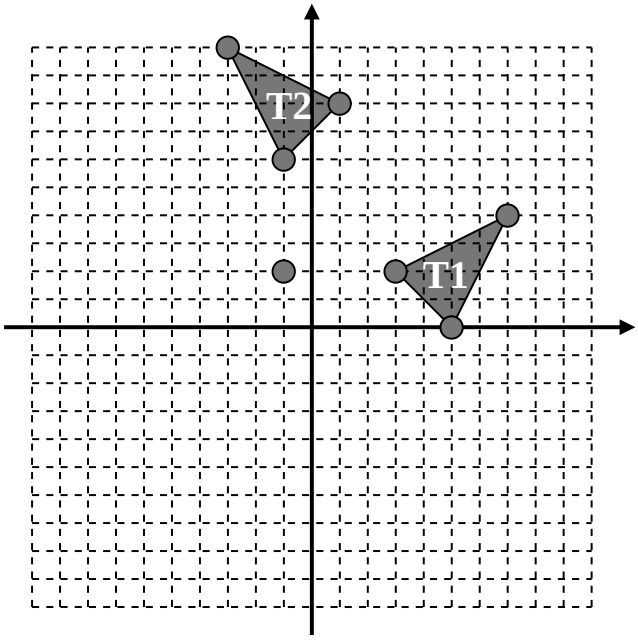
5.4 Answers.

5.4.1 Solution (Example).

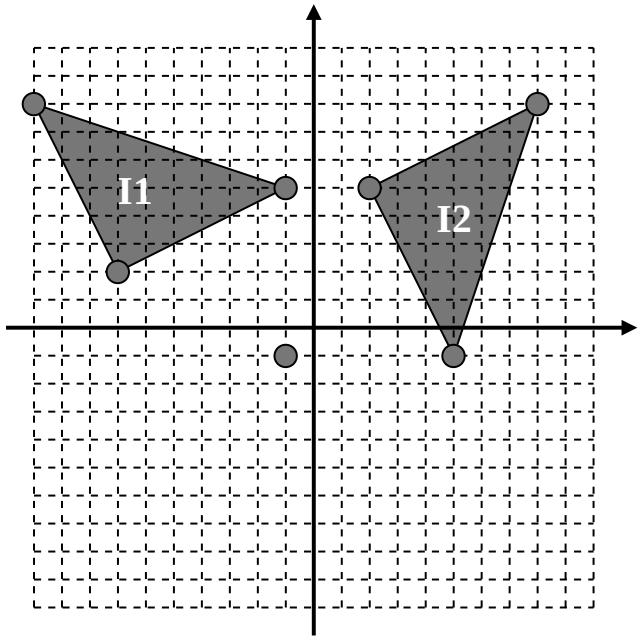


5.4.2. Solution (Exercise).

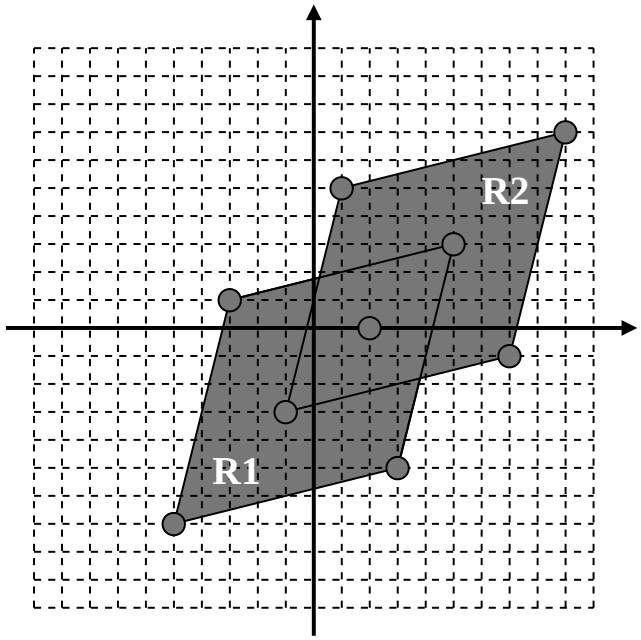
Answer 1.



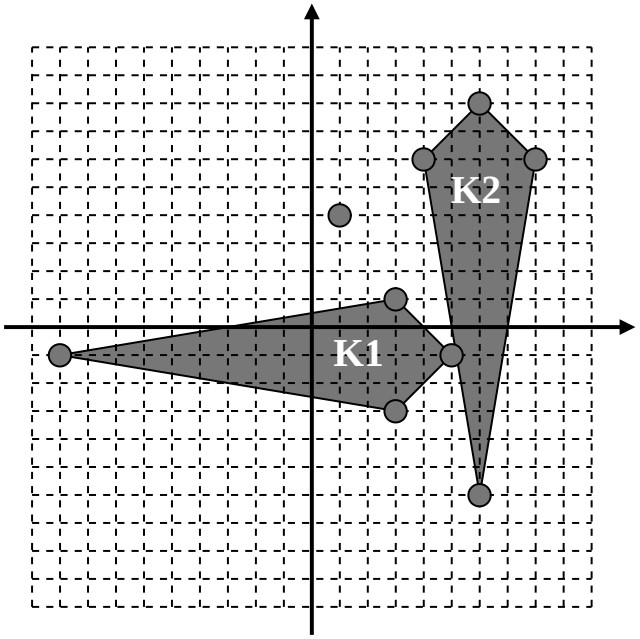
Answer 2.



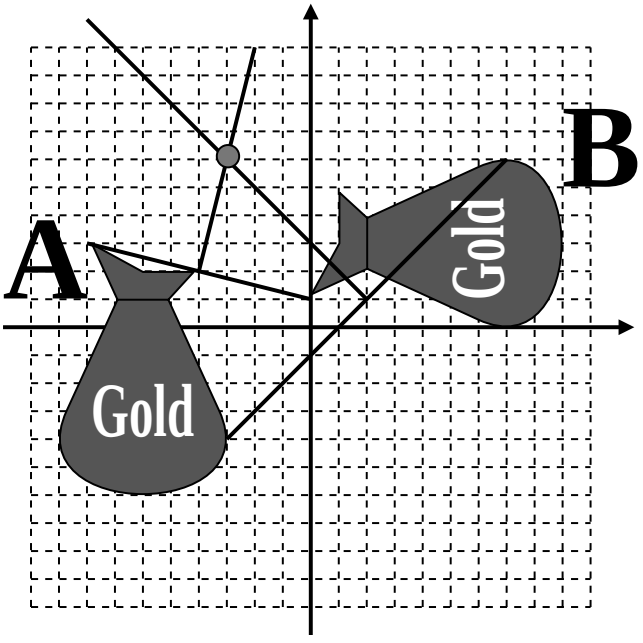
Answer 3.



Answer 4.



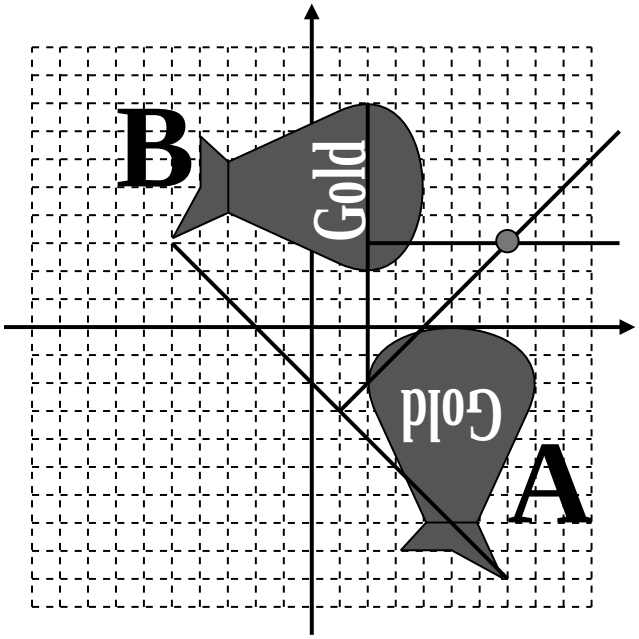
Answer 5.



Rotation of 90° (anti-clockwise) about the point (- 3, 6)

Construction shown but also can be found using tracing paper and experimentation.

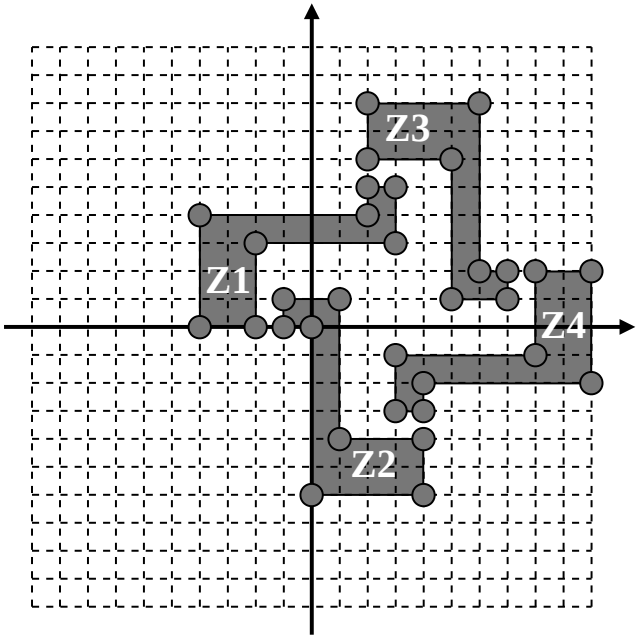
Answer 6.



Rotation of 270° (anti-clockwise) about the point (7, 3)

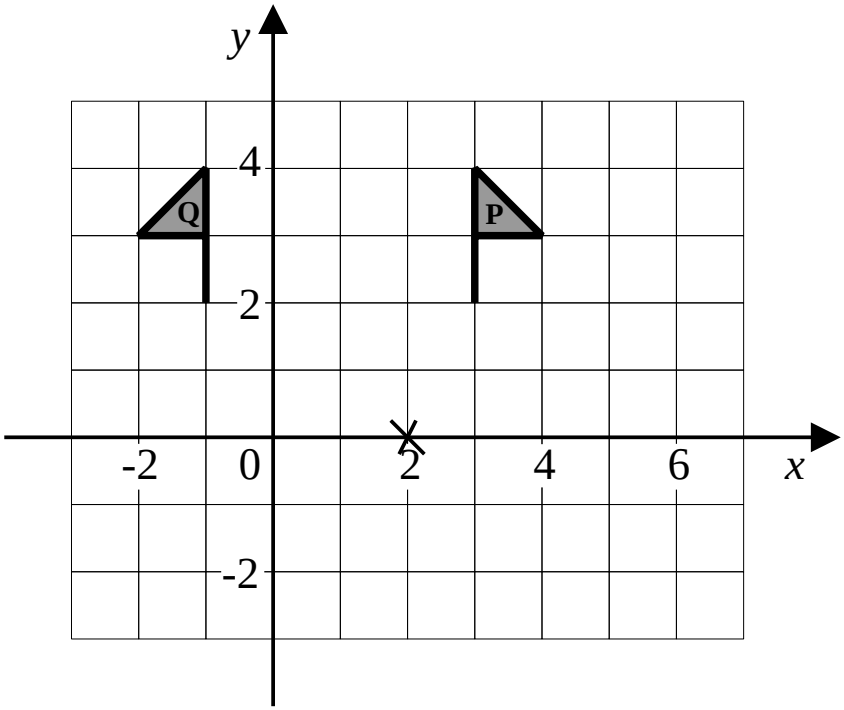
Construction shown but also can be found using tracing paper and experimentation.

Answer 7.



5.5 Homework (Old Examination Questions)

Question 1.
GCSE Examination Question from May 2012 : Q4.



(a) Describe fully the single transformation that maps shape **P** onto shape **Q**.

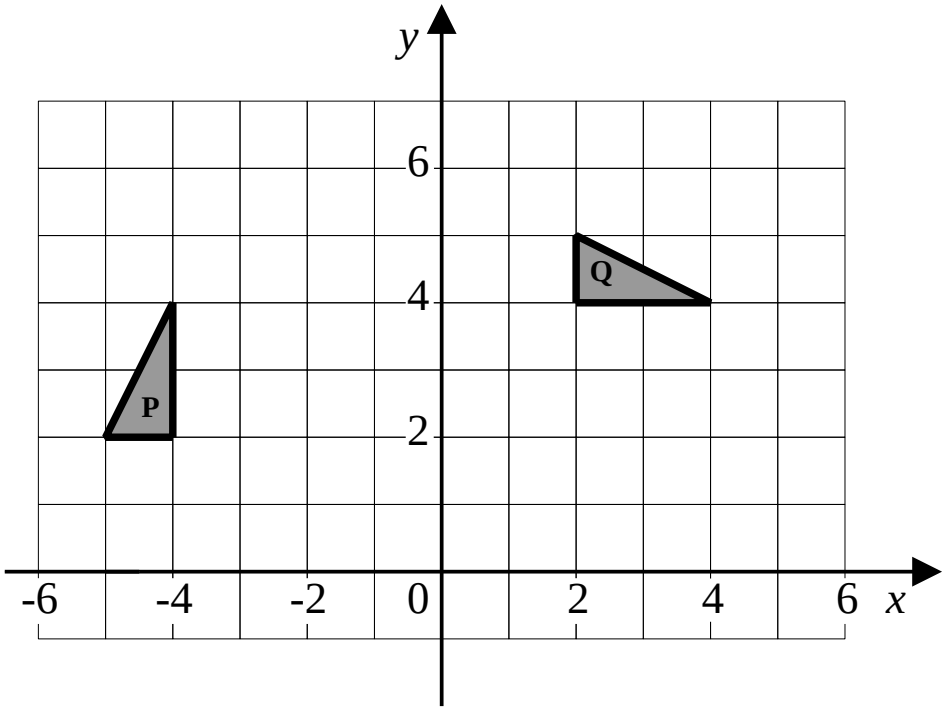
[2 marks]

(b) On the grid, rotate shape **P** 90° clockwise about the point (2, 0).
Label the new shape **R**.

[2 marks]

Question 2.

GCSE Examination Question from June 2011 : Q6.



(a) Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

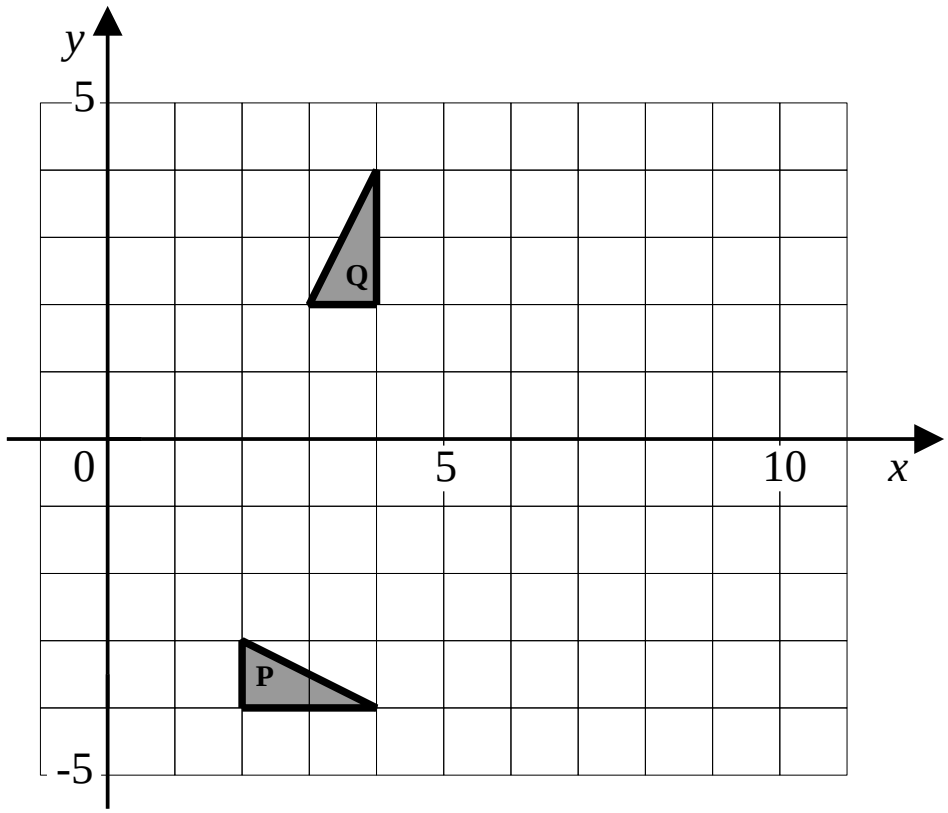
[3 marks]

(b) Reflect triangle **Q** in the line $y = x$.

Label the new triangle **R**.

[2 marks]

Question 3.
GCSE Examination Question from January 2012 : Q12.



(a) Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

[3 marks]

(b) On the grid, translate triangle **Q** by the vector $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

Label the new triangle **R**.

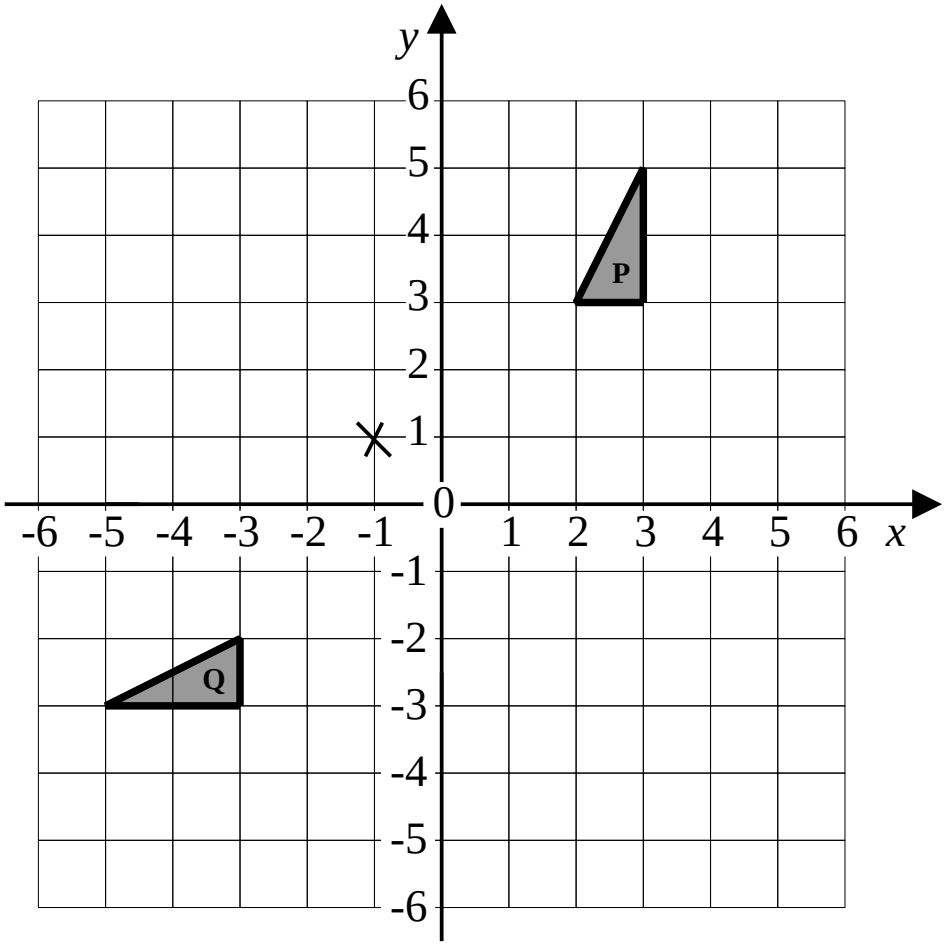
[1 mark]

(c) Describe fully the single transformation that maps triangle **P** onto triangle **R**.

[2 marks]

Question 4.

GCSE Examination Question from June 2010 : Q9.



(a) Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

[2 marks]

(b) Rotate triangle **Q** through 90° anti-clockwise about (-1. 1). Label the new triangle **R**.

[2 marks]

(c) Describe fully the single transformation that maps triangle **P** onto triangle **R**.

[2 marks]

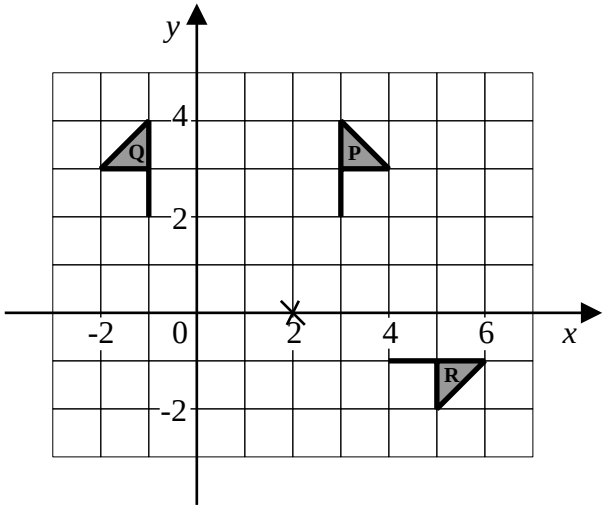
5.6 Answers.

5.6.1 Solutions (5.5 Homework (Old Examination Questions))

Answer 1.

(a) Reflection in the line $x = 1$.

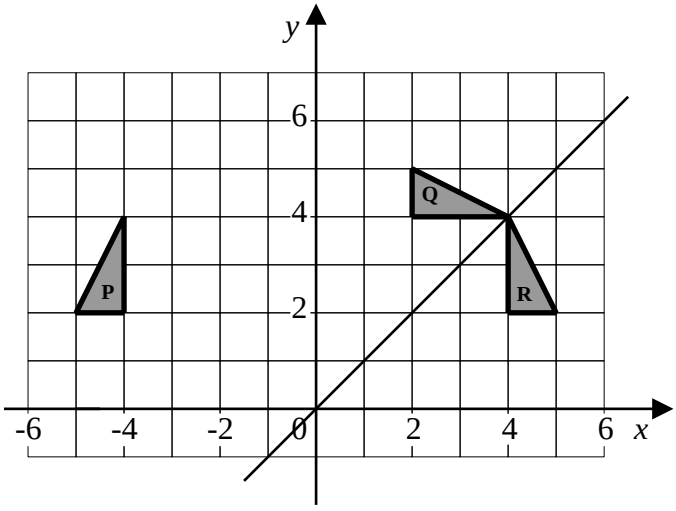
(b)



Answer 2.

(a) Rotation -90° about (0, 0) (Note: - 90° = 90° clockwise)

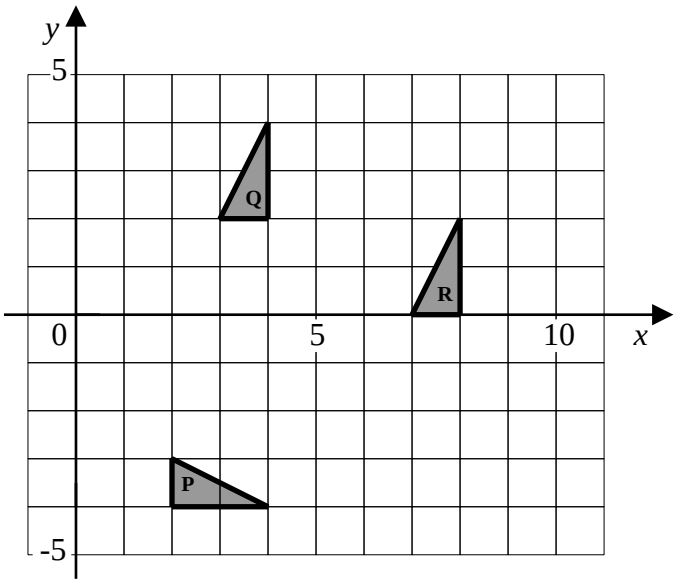
(b)



Answer 3.

(a) Rotation 90° about (0, 0) (Note 90° = 90° anti-clockwise)

(b)

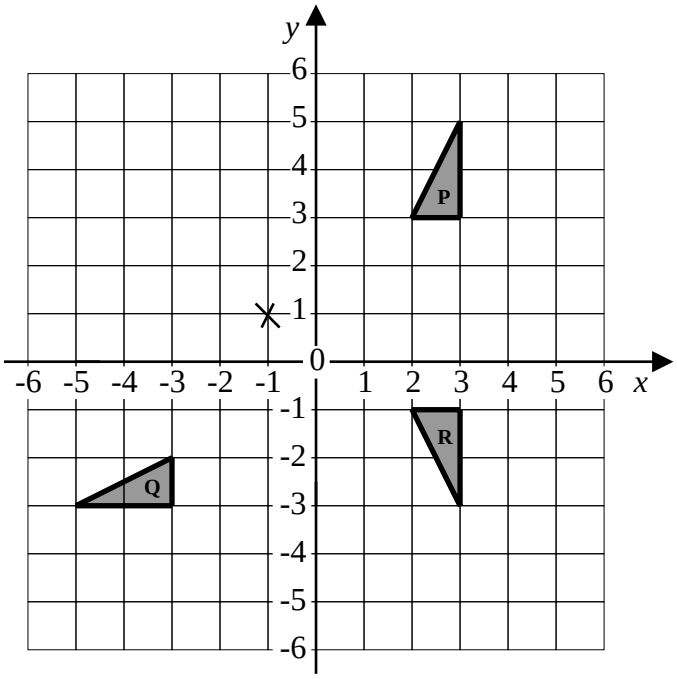


(c) Rotation 90° about (3, 1) (Note 90° = 90° anti-clockwise)

Answer 4.

(a) Reflection in the line $y = -x$

(b)



(c) Reflection in the line $y = 1$.

Chapter 6.

6.1 T4. Enlargement with positive scale factor.

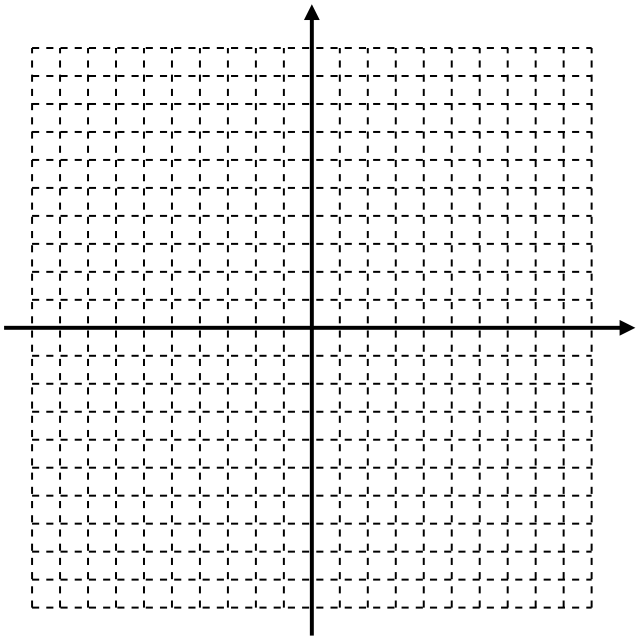
An ***enlargement*** is the fourth of the four ***transformations*** to be studied.

Two items of information describe an enlargement fully:

En1. The **scale factor** of the enlargement.

En2. The **centre** (a point, the focus) of the enlargement.

6.2 Example.



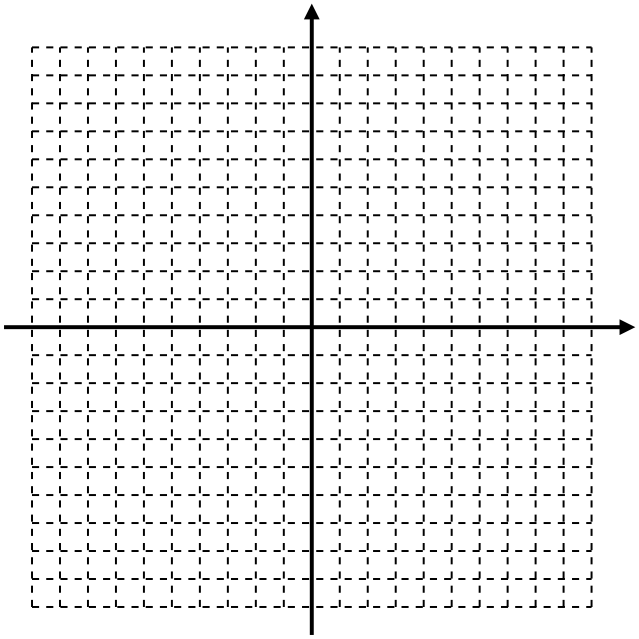
(i) Plot the X-shape, *X*, with vertices;

A (3, 8),	*B* (1, 7),	*C* (- 1, 8),	*D* (- 1, 7),
E (0, 6),	*F* (- 1, 5),	*G* (- 1, 4),	*H* (1, 5),
I (3, 4),	*J* (3, 5)	*K* (2, 6),	*L* (3, 7).

(ii) Enlarge *X* with scale factor 3, centre (2, 10).

6.3 Exercise.

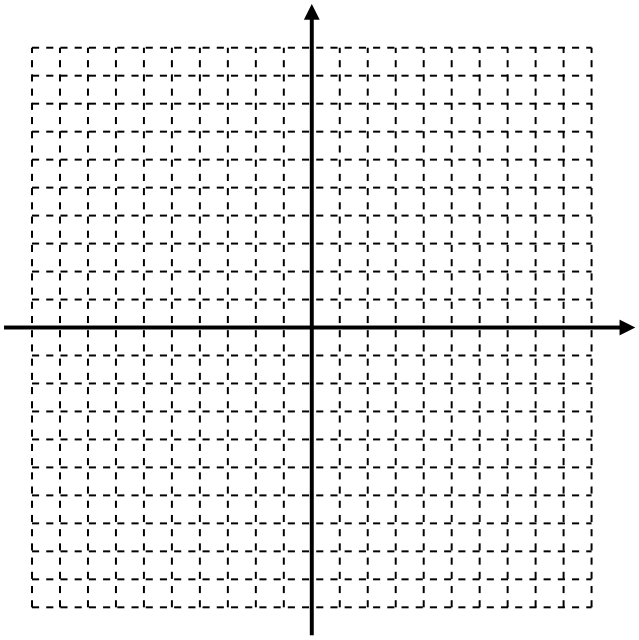
Question 1.



(i) Plot the six sided shape, *H*, with vertices;
U (10, 4), *V* (8, 6), *W* (8, 5),
X (5, 5), *Y* (7, 3), *Z* (7, 4).

(ii) Enlarge *H* with scale factor 4, centre (10, 7).

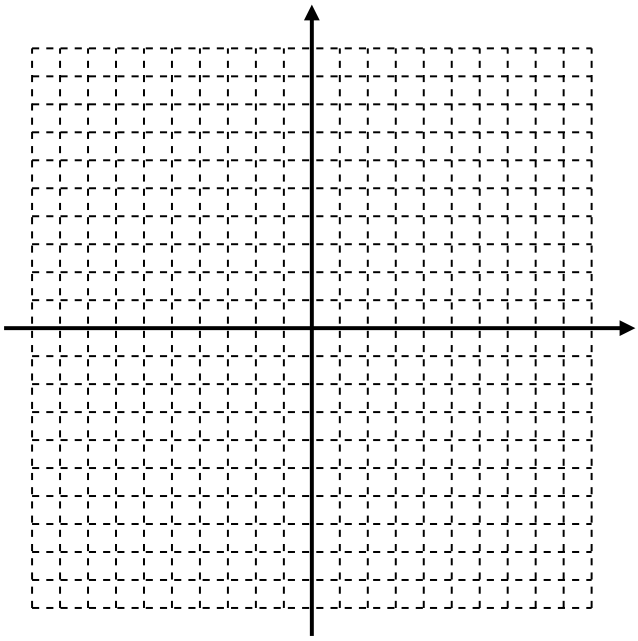
Question 2.



(i) Plot the space invader, *SI*, with vertices;
P (10, 0), *Q* (4, 6), *R* (4, 3), *S* (- 2, 3),
T (- 2, 6), *U* (- 8, 0), V (1, - 3).

(ii) Enlarge *SI* with scale factor "one third", centre (7, - 9).

Question 3.

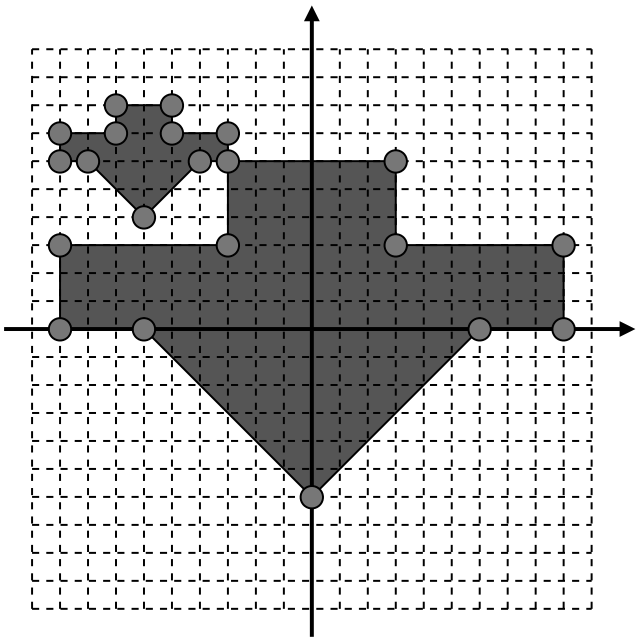


(i) Plot the vase, *V*, with vertices;

A (4, 1), *B* (3, 2), *C* (3, 3), *D* (0, 3),
E (0, 2), *F* (- 1, 1), *G* (1, - 1), *H* (2, - 1).

(ii) Enlarge *V* with scale factor 3, centre (1, 1).

Question 4.

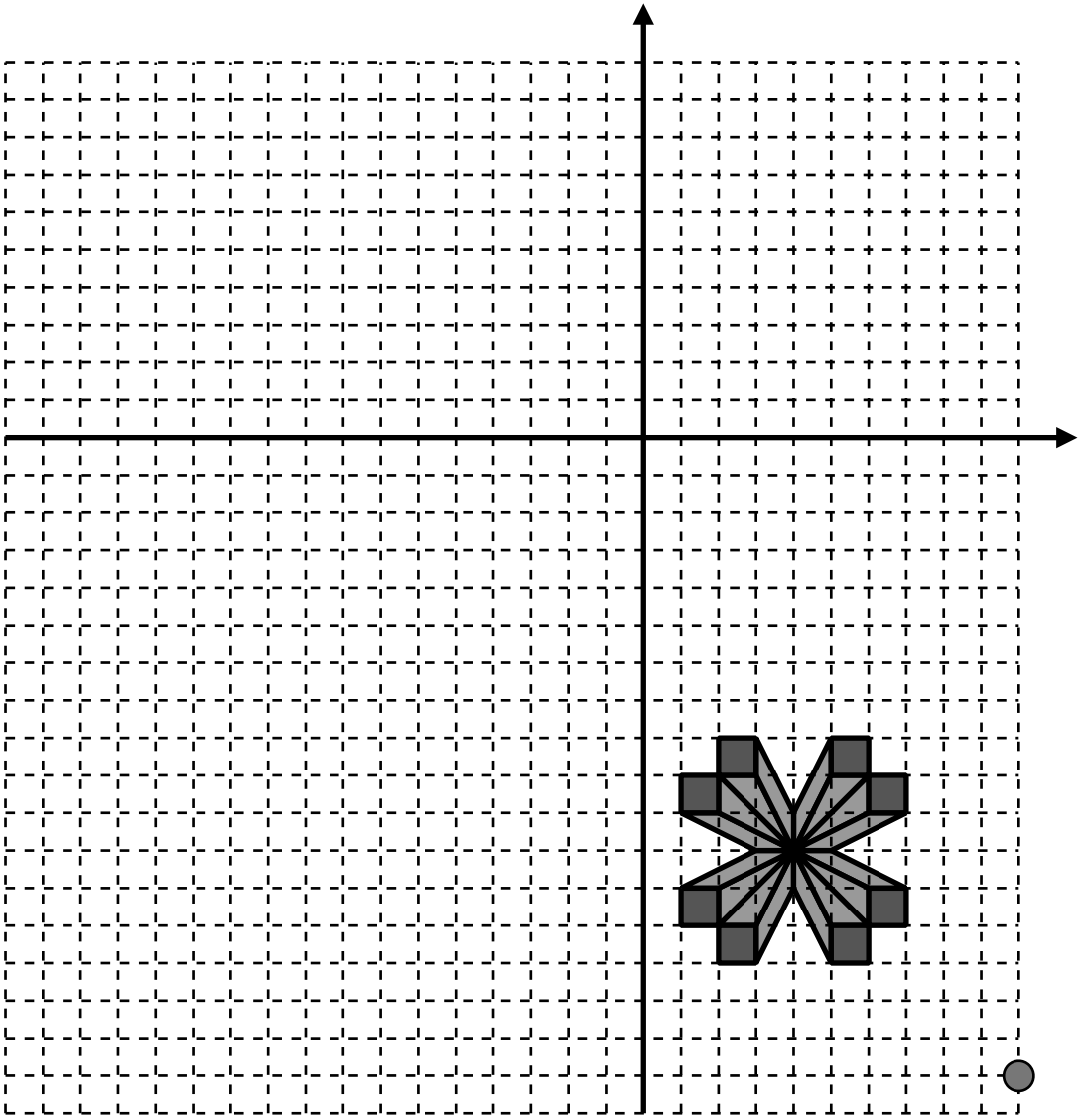


The small 'spinning top' is enlarged onto the big.

(i) What is the scale factor of the enlargement ?

(ii) Where is the centre of the enlargement ?

Question 5.



Enlarge the given geometric shape by scale factor 3 and centre (10, - 17)

Question 6.

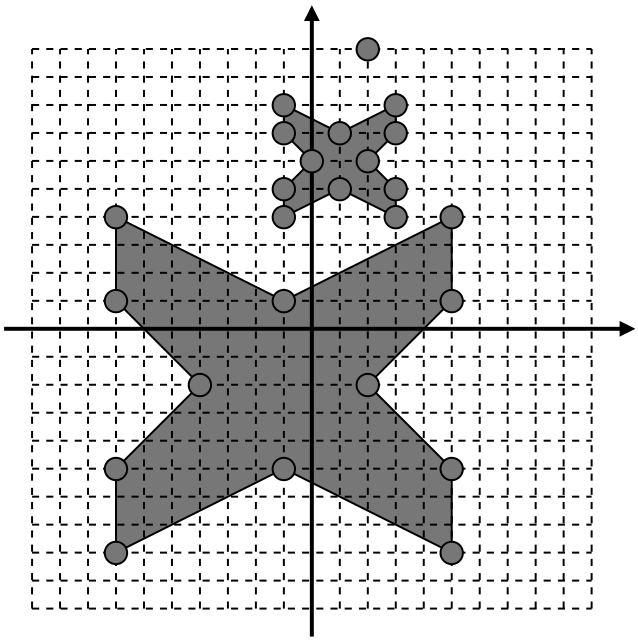
Wolverhampton has a football team (The 'Wolves') which has a distinctive badge in the form of a black wolf face on a yellow background.

Use the internet to find this badge.
Copy it onto squared paper.

Now enlarge your copy, making it three times as big and clearly marking the centre of your enlargement on the squared paper.

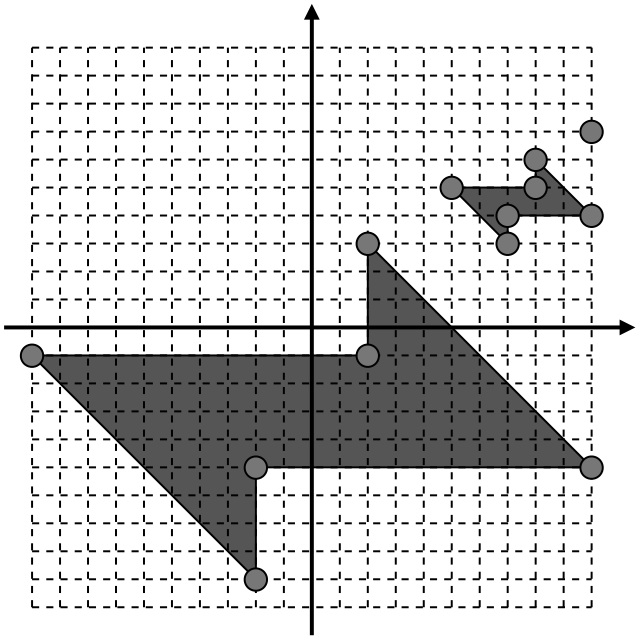
6.4 Answers.

6.4.1 Solution (Example).

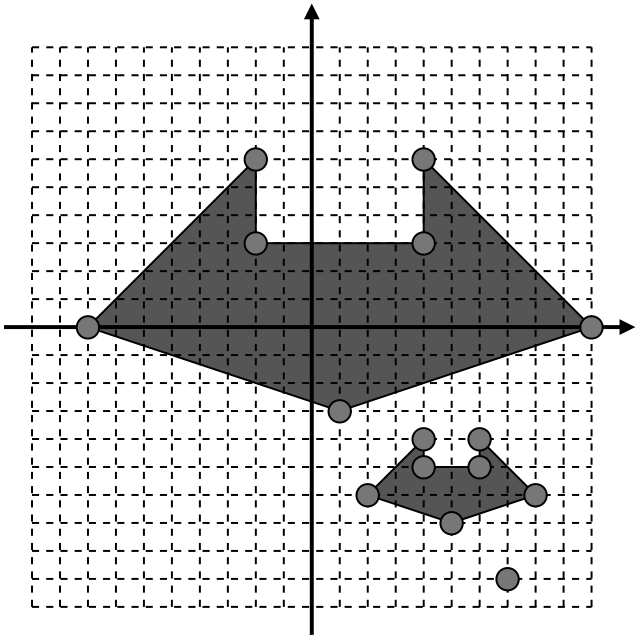


6.4.2 Solution (Exercise).

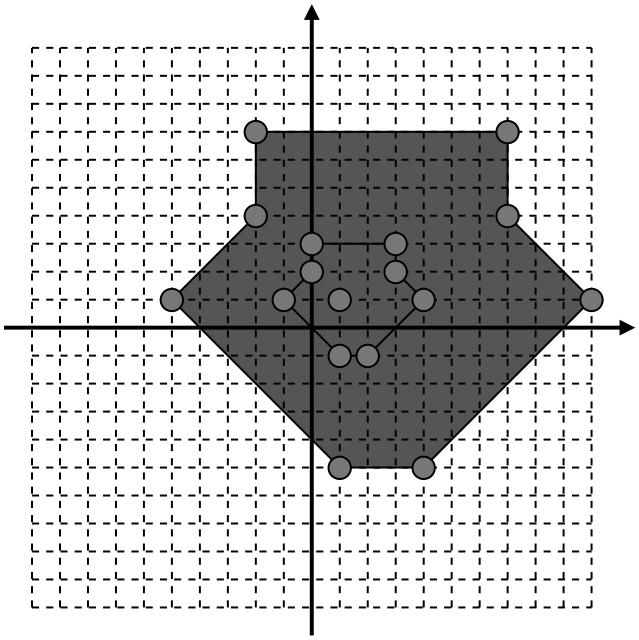
Answer 1.



Answer 2.



Answer 3.

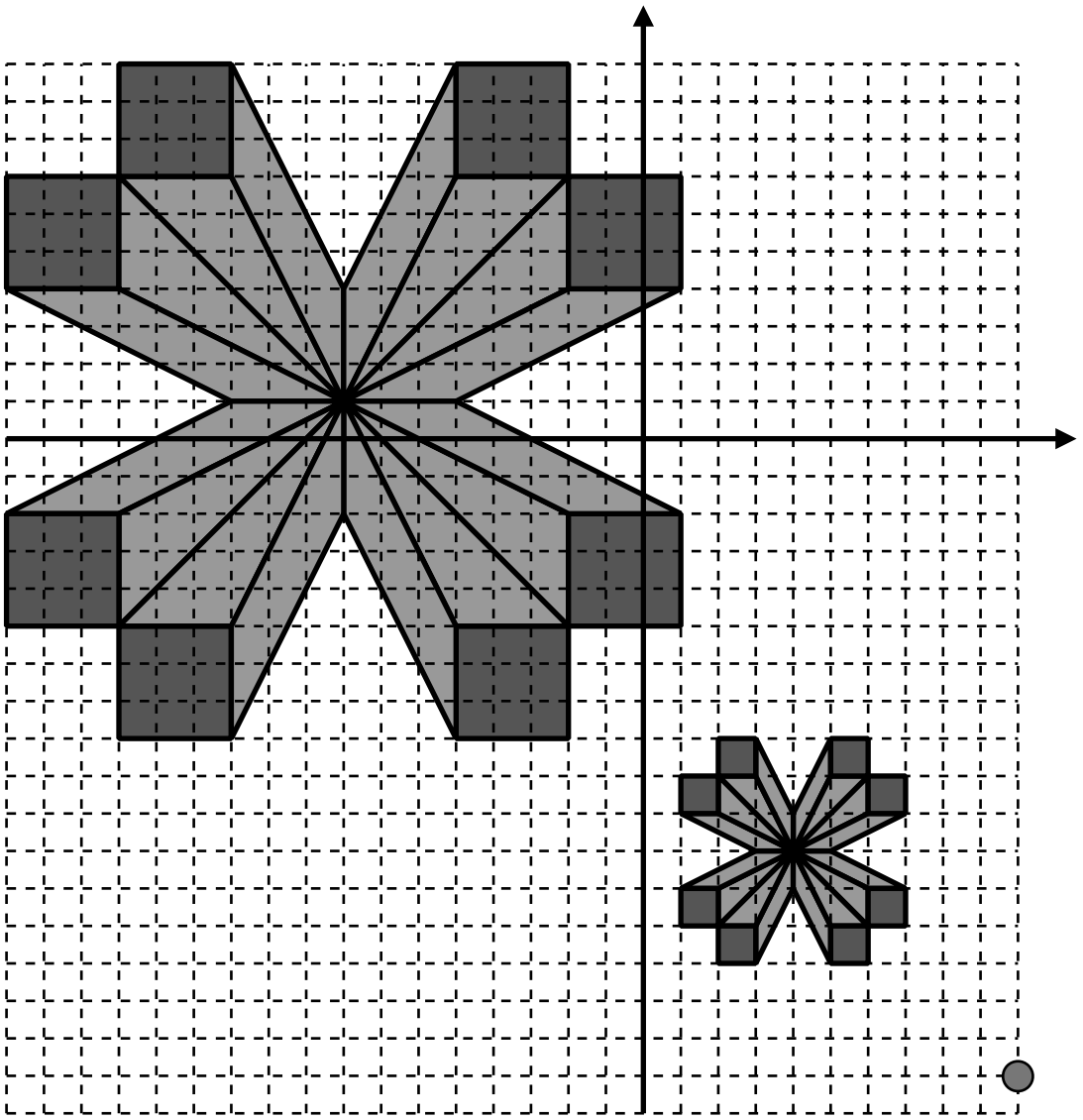


Answer 4.

(i) 3

(ii) (- 9, 9)

Answer 5.



Question 6.

Mark as appropriate.

Chapter 7.

7.1 T4. Enlargement with negative scale factor.

In chapter 6 it was observed that what mathematicians call an enlargement can sometimes result in a smaller shape.

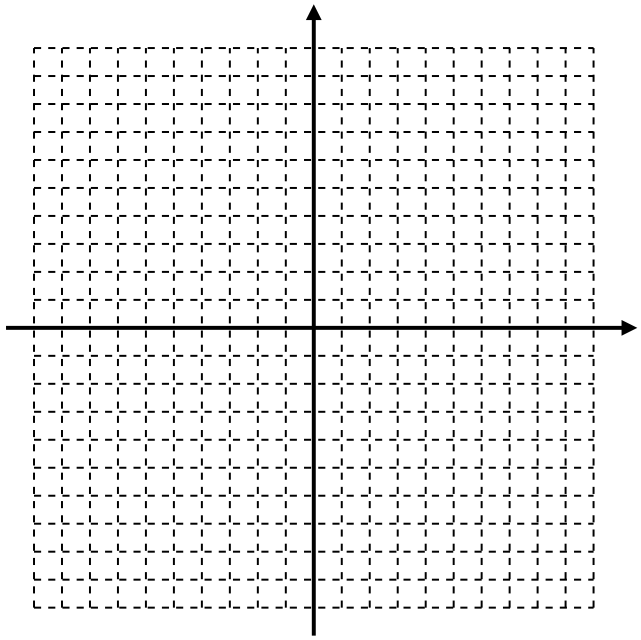
SF1. IF ***scale factor*** > 1 THEN a bigger, similar shape results.
SF2. IF 0 < ***scale factor*** < 1 THEN a smaller, similar shape results.

7.2.1 Example.

What happens if **(a)** ***scale factor*** = 1 ?

(b) ***scale factor*** = 0 ?

7.2.2 Example.



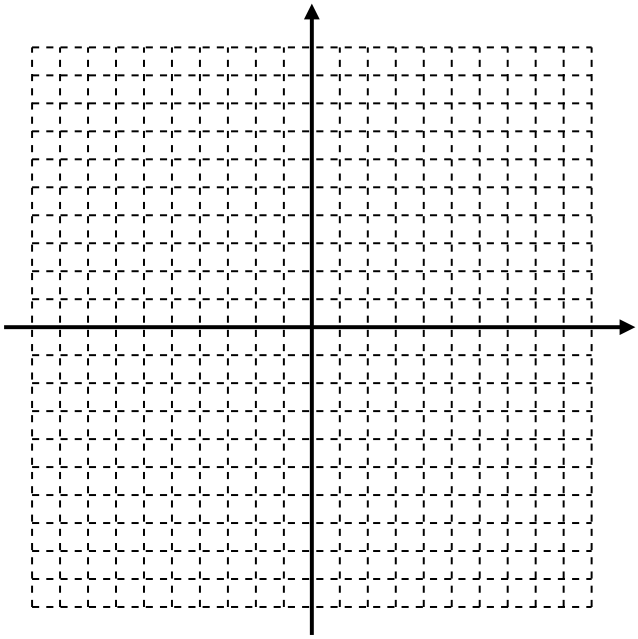
(a) Plot the ARROW-shape with vertices;

A (3, 3),	*B* (6, 0),	*C* (9, 3),	*D* (7, 3),
E (7, 9),	*F* (3, 9),	*G* (3, 7),	*H* (5, 7),
I (5, 3).			

(b) Enlarge the ARROW with scale factor - 2 and centre (3, 3).

7.3 Exercise.

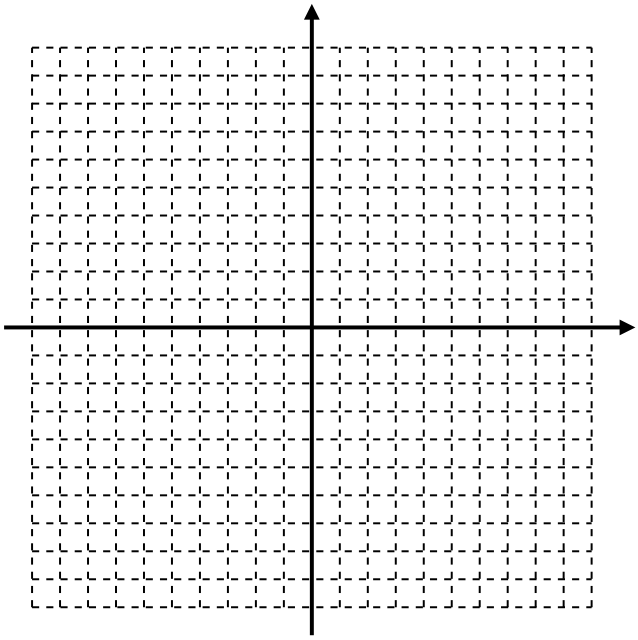
Question 1.



(a) Plot the FLAG with vertices;
Q (6, 8), R (6, 2), S (7, 2),
T (7, 6), U (9, 6), V (9, 8).

(b) Enlarged FLAG with scale factor - 3, centre (5, 4).

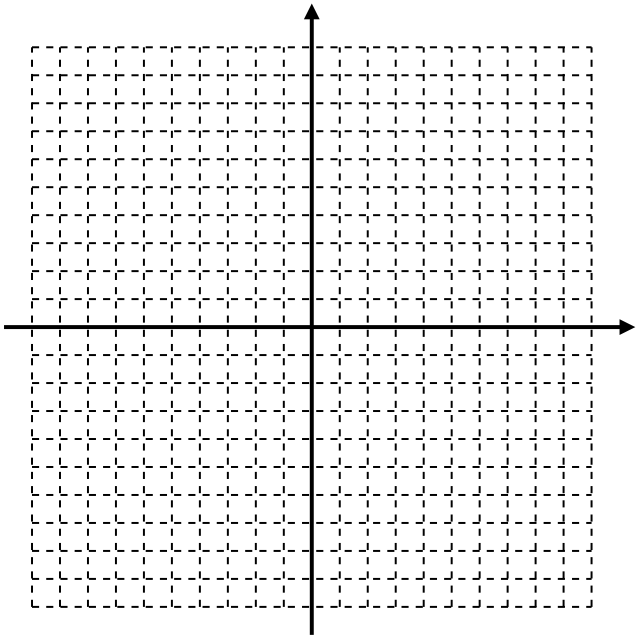
Question 2.



(a) Plot (carefully!) the BUZZ-SAW shape with vertices;
S (- 7, 7) T (- 8, 8) U (- 8, 9), V (- 9, 8),
W (- 10, 8), X (- 9, 7), Y (- 9, 6), Z (- 8, 7).

(b) Enlarge the BUZZ-SAW by scale factor - 3, centre (-5, 5).

Question 3.



(a) Plot the CROSS shape with vertices;

A (- 1, 2),	*B* (- 1, 5),	*C* (- 4, 5),	*D* (- 4, 8),
E (- 7, 8),	*F* (- 7, 5),	*G* (- 10, 5),	*H* (- 10, 2),
I (- 7, 2),	*J* (- 7, - 7)	*K* (- 4, - 7),	*L* (- 4, 2).

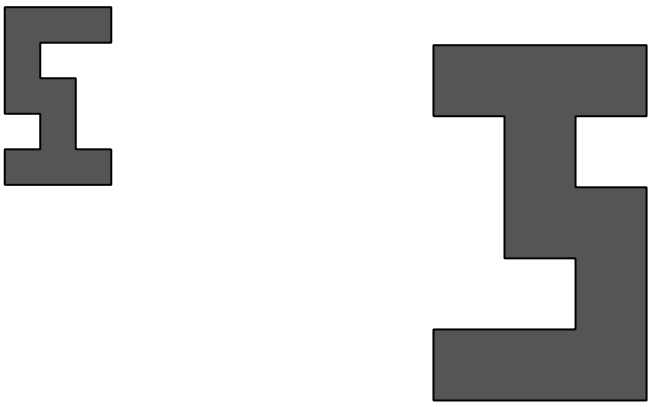
(b) Label your cross *X*1.

(c) Enlarge the CROSS by scale factor - 1, centre (- 1, 0).

(d) Label the enlargement *X*2.

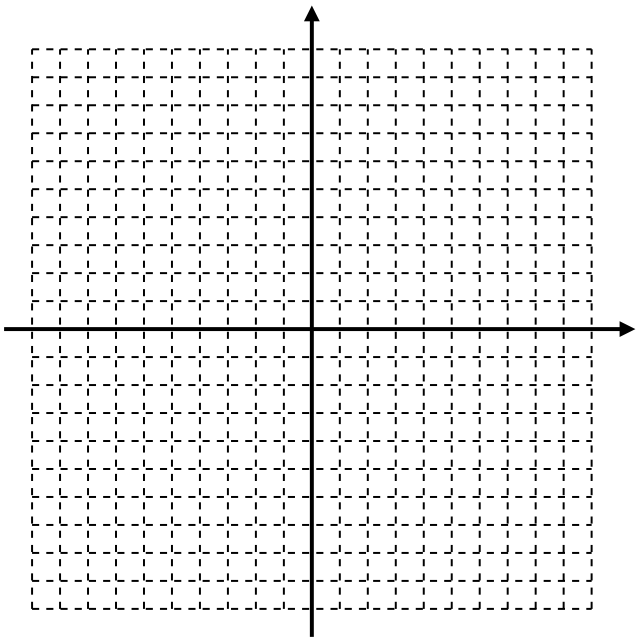
(e) What other transformation would move *X*1 to *X*2 ?

Question 4.



Use a straight edge to draw lines that show where the centre of the enlargement is.

Question 5.



(a) Plot the DART shape with vertices;

H (9, 0), I (1, 4), J (- 3, 8), K (- 9, 6),
L (- 3, 0), M (- 9, - 5), N (- 3, - 8), O (1, - 4).

(b) Enlarge the DART by scale factor - 0.5, centre (3, 4).

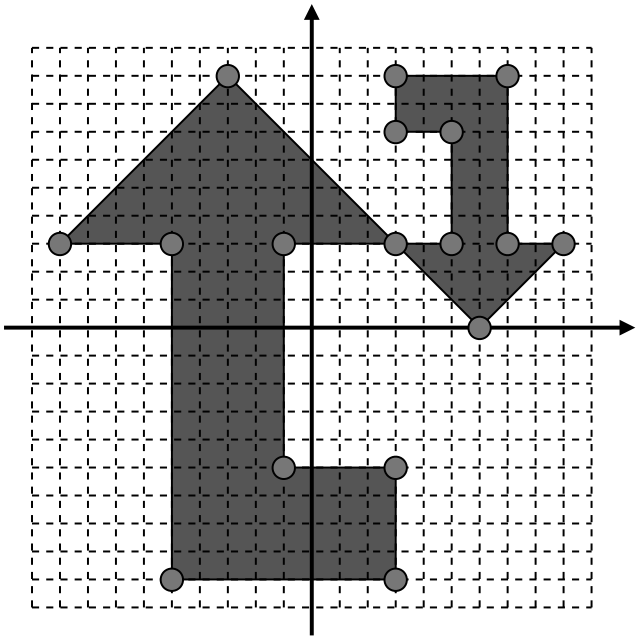
7.4 Answers.

7.4.1 Solution (Example).

(a) The image is congruent to the original shape and in the same location.

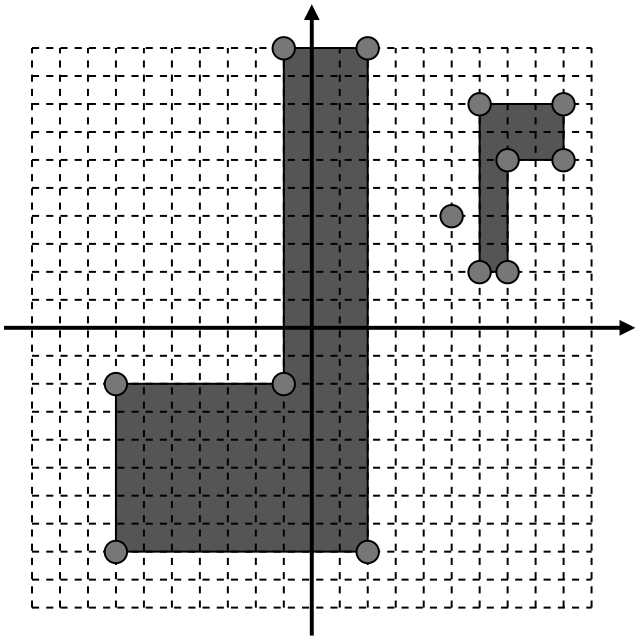
(b) There is no image. (It's collapsed into a singularity)

7.4.2 Solution (Example).

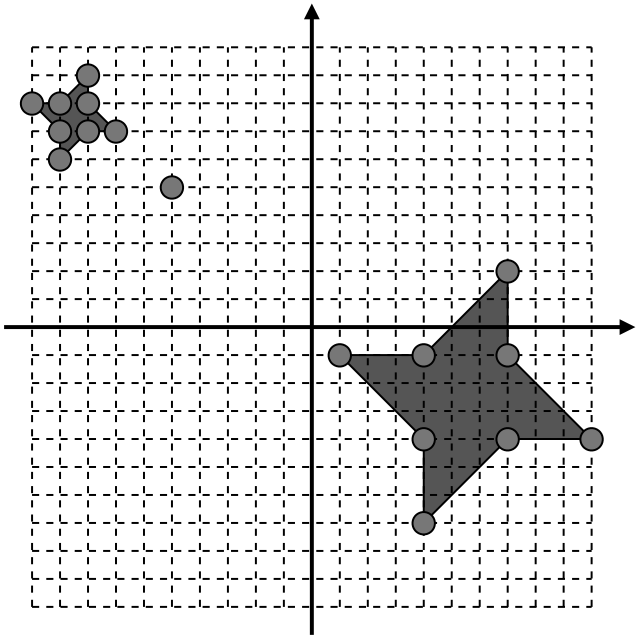


7.4.3 Solution (Exercise).

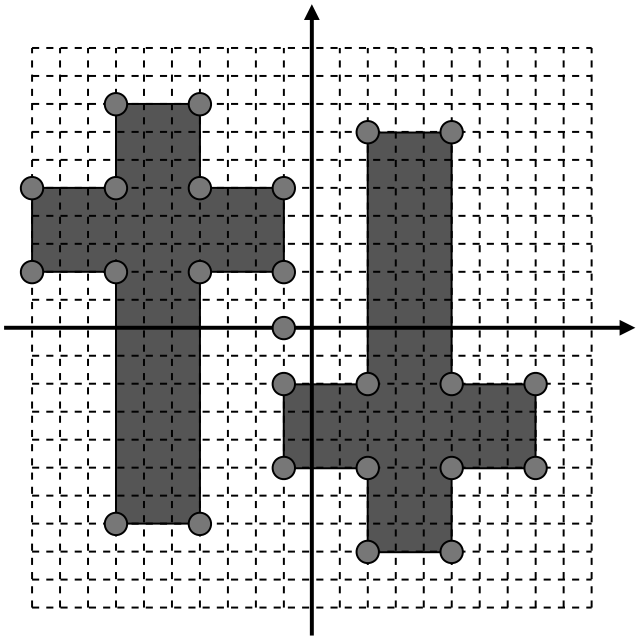
Answer 1.



Answer 2.

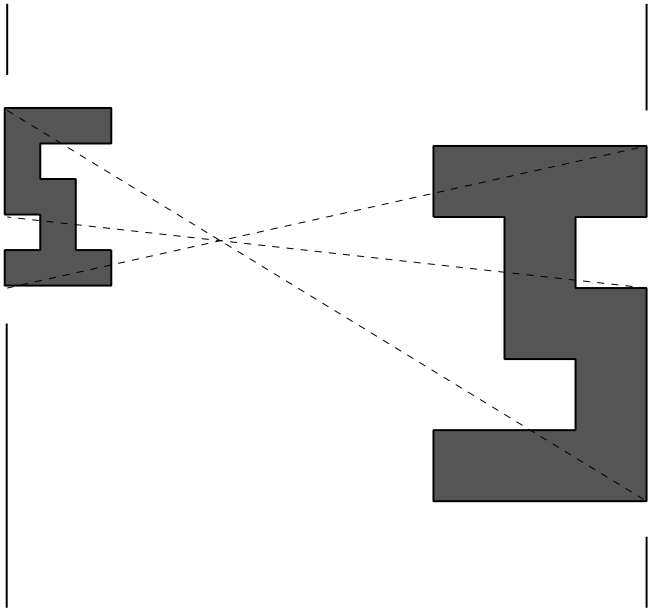


Answer 3.

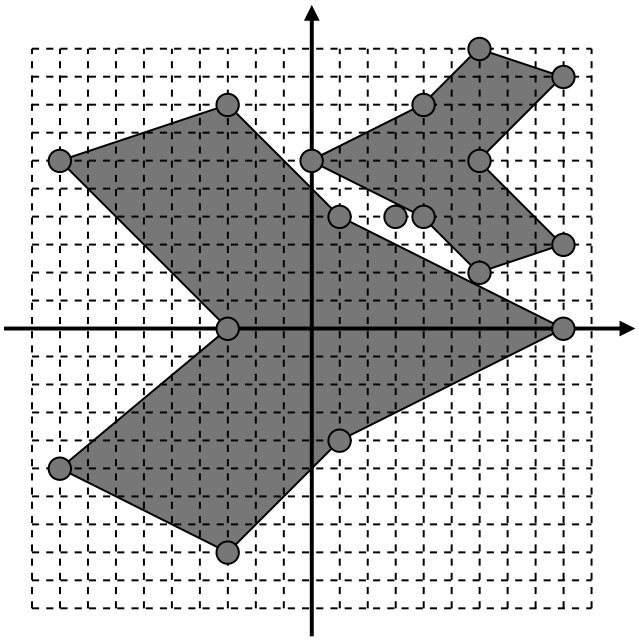


(e) Rotation 180° about (-1, 0)

Answer 4.

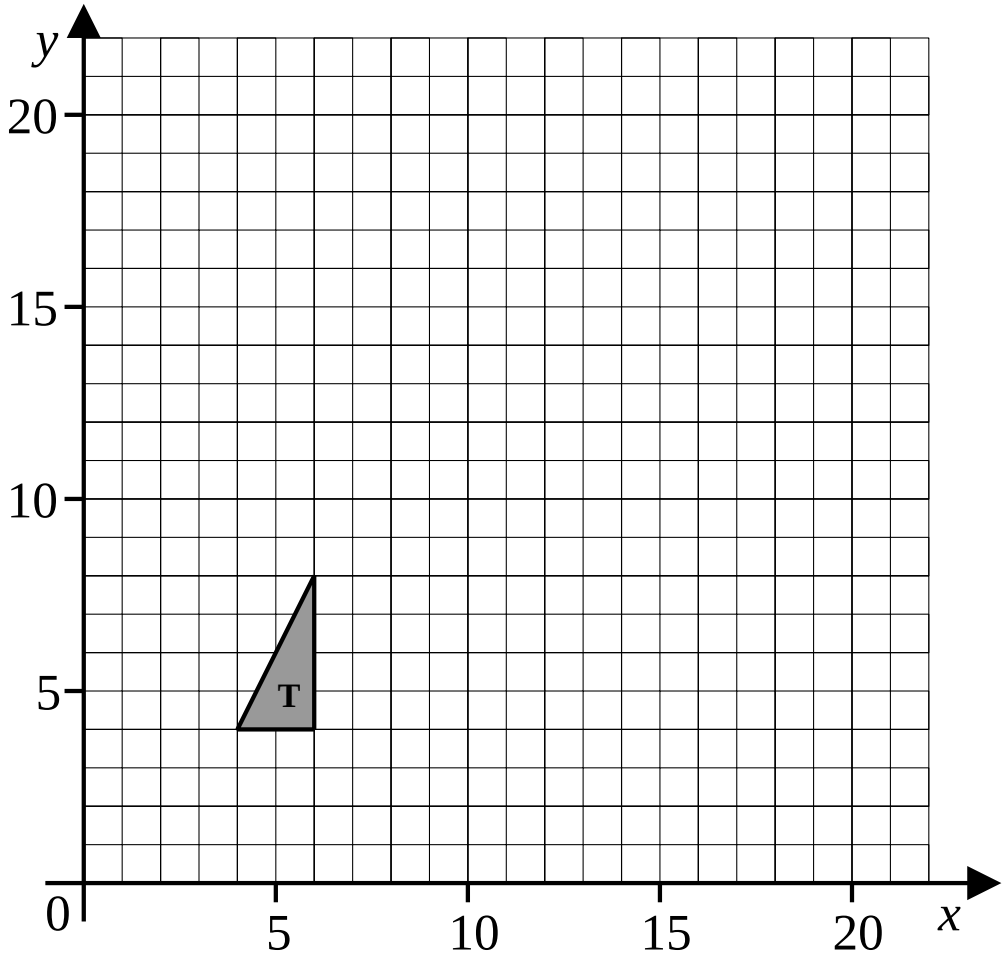


Answer 5.



7.5 Homework (Old Examination Questions)

Question 1.
GCSE Examination Question from May 2009 : Q3.



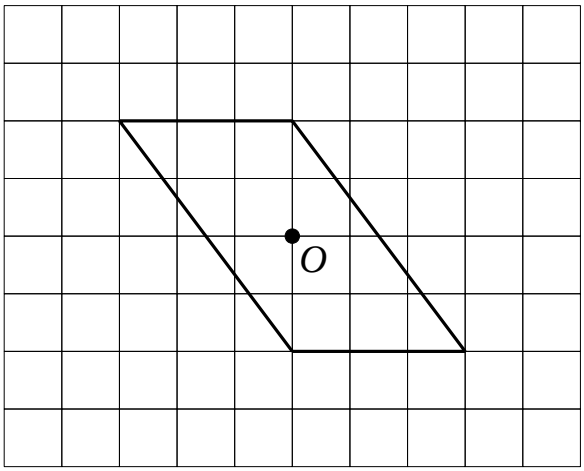
On the grid, enlarge triangle **T** with a scale factor of $2\frac{1}{2}$ and centre (0, 0)

[3 marks]

Question 2.
GCSE Examination Question from June 2009 : Q7.

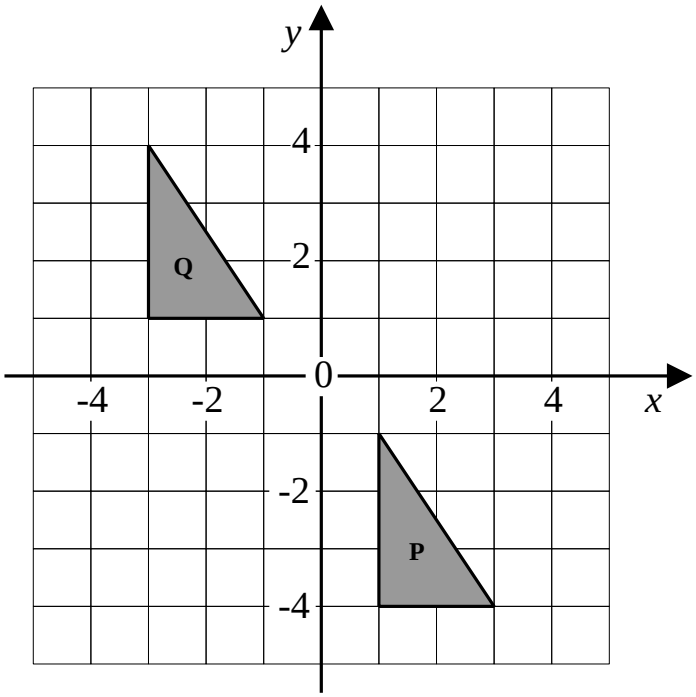
The diagram shows a parallelogram.

(a) On the grid, rotate the parallelogram through 90° anticlockwise about the point *O*.



[2 marks]

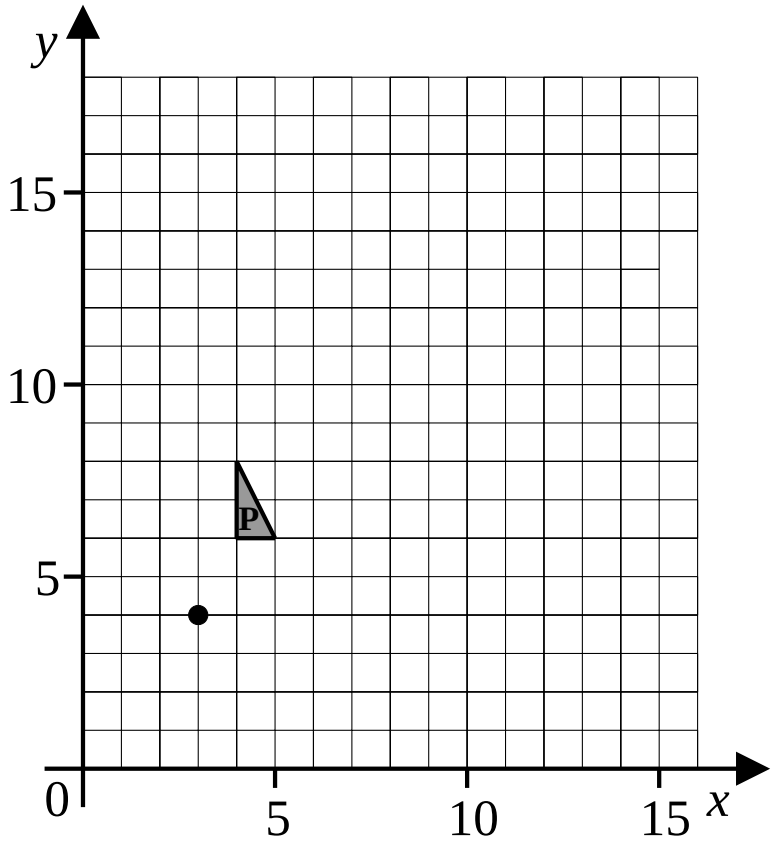
(b)



Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

[2 marks]

Question 3.
GCSE Examination Question from November 2010 : Q7.



(a) On the grid, enlarge triangle **P** with scale factor 3 and centre (3, 4).
Label the new triangle **Q**.

[3 marks]

(b) On the grid, translate triangle **Q** by the vector $\begin{bmatrix} 4 \\ -8 \end{bmatrix}$.
Label the new triangle **R**.

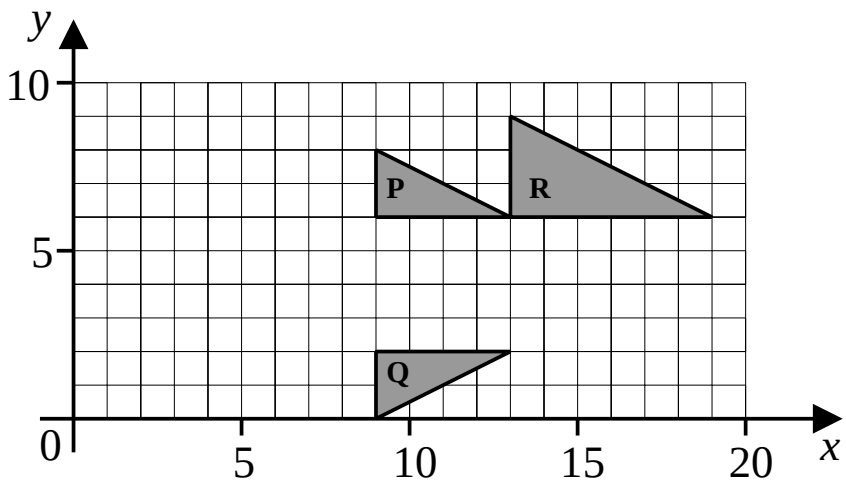
[2 marks]

(c) Describe fully the single transformation which
maps triangle **P** onto triangle **R**.

[2 marks]

Question 4.

GCSE Examination Question from November 2009 : Q6.



(a) Describe fully the single transformation which maps triangle **P** onto triangle **Q**.

[2 marks]

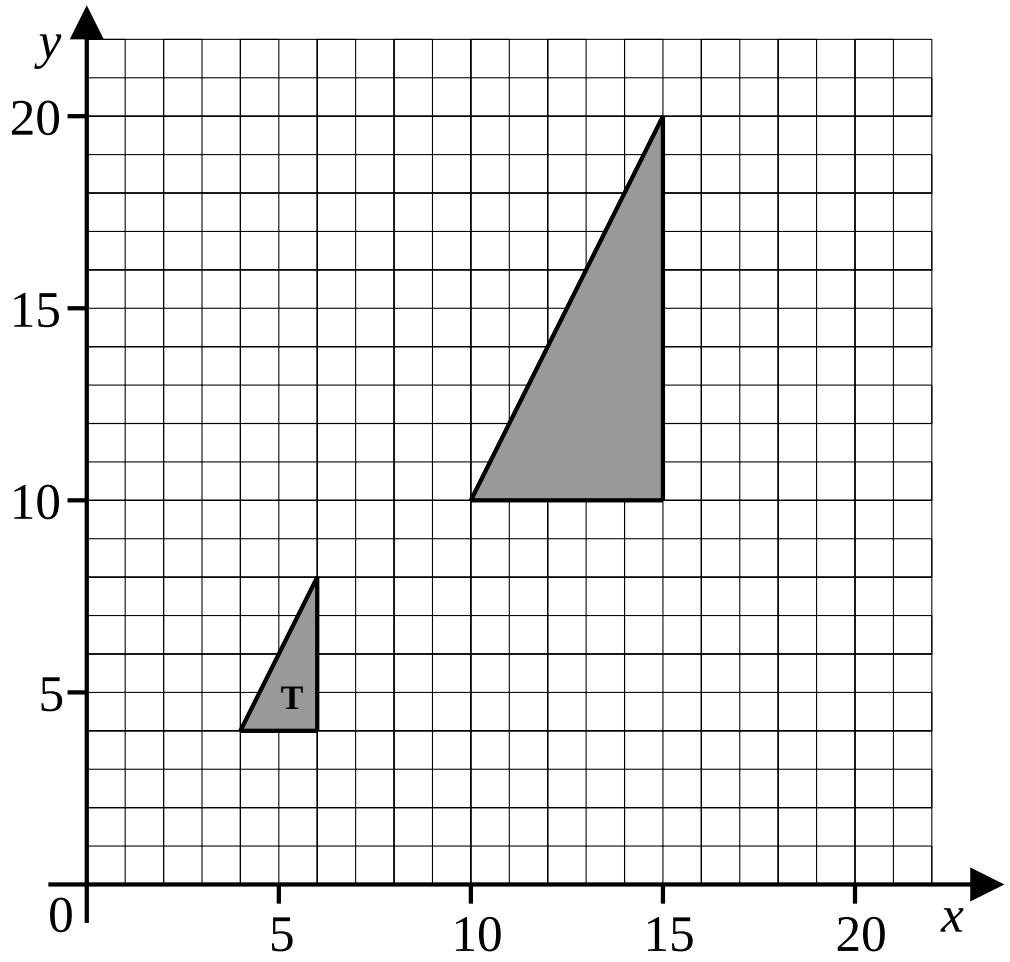
(b) Describe fully the single transformation which maps triangle **P** onto triangle **R**.

[3 marks]

7.6 Answers.

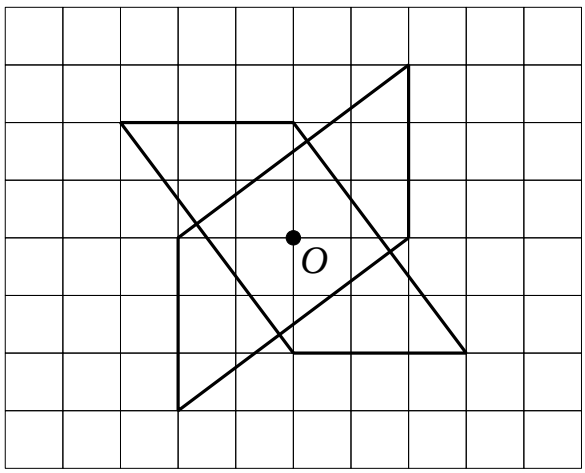
7.6.1 Solutions (7.5 Homework (Old Examination Questions))

Answer 1.



Answer 2.

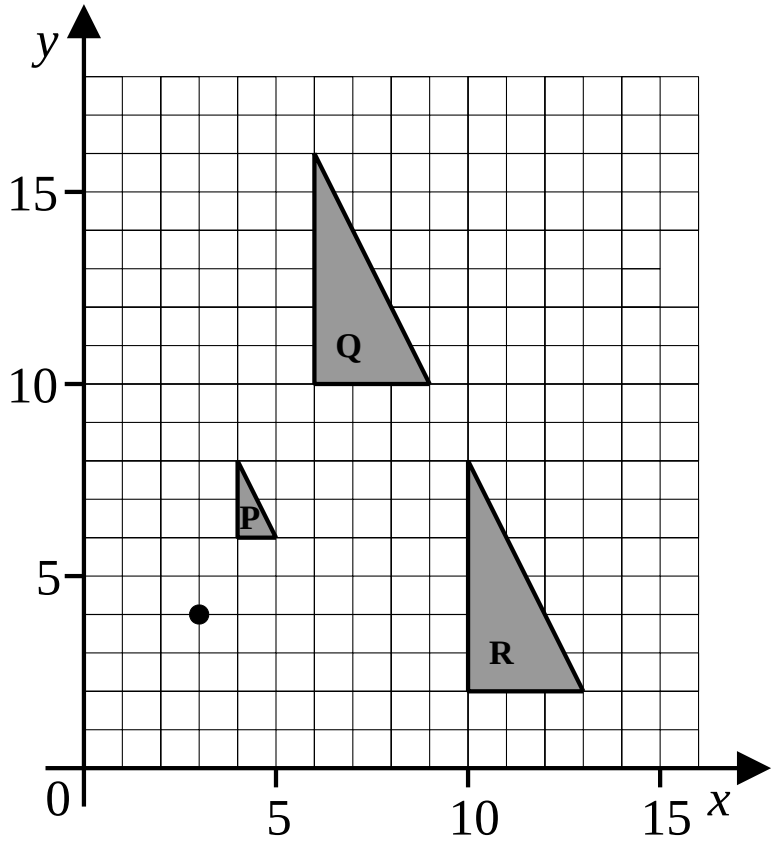
(a)



(b) Translation of $\begin{bmatrix} -4 \\ 8 \end{bmatrix}$.

Answer 3.

(a) & (b)



(c) Enlargement, scale factor 3, centre (1, 8)

Answer 4.

(a) Reflection in the line $y = 4$

(b) Enlargement, scale factor $1\frac{1}{2}$, centre (1, 6)

Chapter 8.

8.1 Summary.

This presentation of ***transformation geometry*** highlighted:

T1. Translation.
T2. Reflection.
T3. Rotation.
T4. Enlargement.

A transformation acts on an original shape, which is transformed to give an ***image***.
Note that the 'shape' can be a 'point' or a 'line'.

8.1.1 T1. Translation.

A ***translation*** is a 'slide' of a fixed shape across the page without rotation.

Translations are described by ***vectors*** such as $\begin{bmatrix} 5 \\ 9 \end{bmatrix}$. (Go 5 across and 9 up)

If one shape is a translation of another then the two shapes are ***directly congruent***.
Congruent means having the same size and shape.

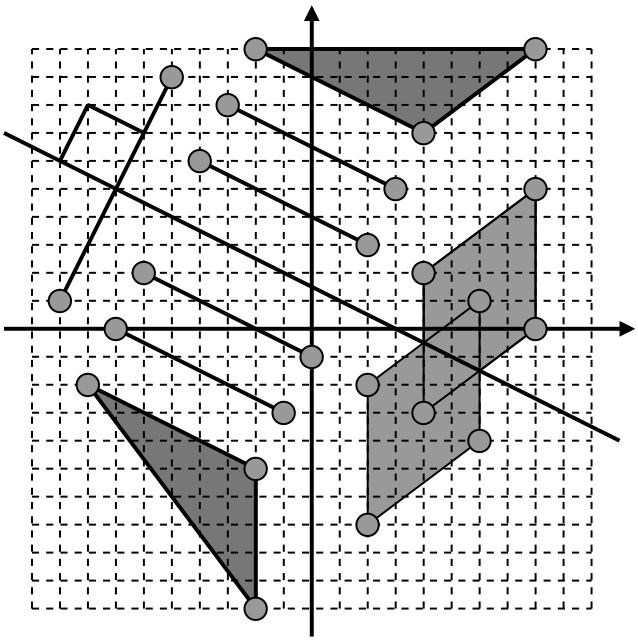
8.1.2 T2. Reflection.

There are three key properties to keep in mind regarding reflections:

Ref 1. The perpendicular distance of any point from the mirror is equal to the perpendicular distance of the image from the mirror.

Ref 2. A line drawn from any point to its image passes through the mirror at 90°.

Ref 3. A line parallel to the mirror has a reflection that is also parallel to the mirror.



The mirror, being a straight line, has an equation of the form $y = m\,x + c$.

8.1.3 T3. Rotation.

Three items of information describe a rotation fully:

Ro1. The size of the **angle** of rotation.

Ro2. The **point** about which rotation takes place.

Ro3. The **direction** of the rotation: clockwise or anti-clockwise.

If the direction is omitted then it is assumed to be anti-clockwise.
In mathematics anti-clockwise is considered to be positive.

8.1.4 T4. Enlargement.

Two items of information describe an enlargement fully:

En1. The **scale factor** of the enlargement.

En2. The **centre** (a point, a focus) of the enlargement.

The scale factor has the following properties;

SF1. IF ***scale factor*** > 1 THEN a bigger, similar shape results.

SF2. IF $0 <$ ***scale factor*** < 1 THEN a smaller, similar shape results.

SF3. IF $-1 <$ ***scale factor*** < 0 THEN a smaller, similar, inverted shape results.

SF4. IF ***scale factor*** < -1 THEN a bigger, similar, inverted shape results.

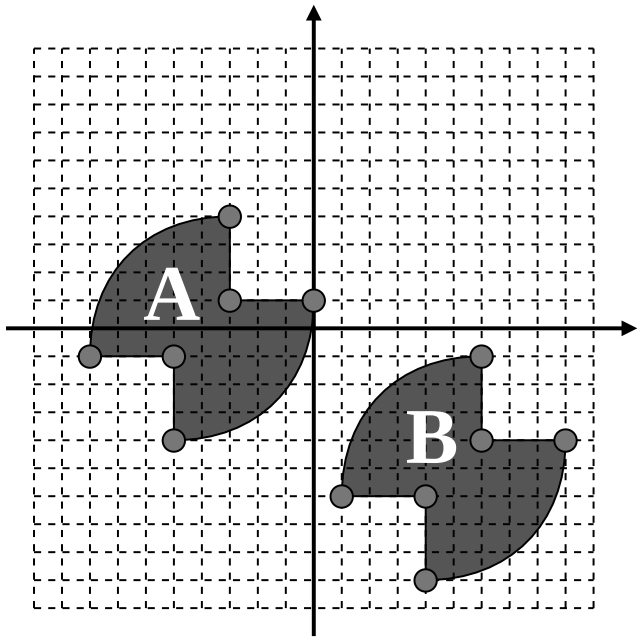
SF5. IF ***scale factor*** $= 1$ THEN the original and image are 'one on top of the other'.

SF6. IF ***scale factor*** $= 0$ THEN there is no image.

SF7. IF ***scale factor*** $= -1$ THEN the image is a rotation of 180° about the focus.

8.2 Exercise.

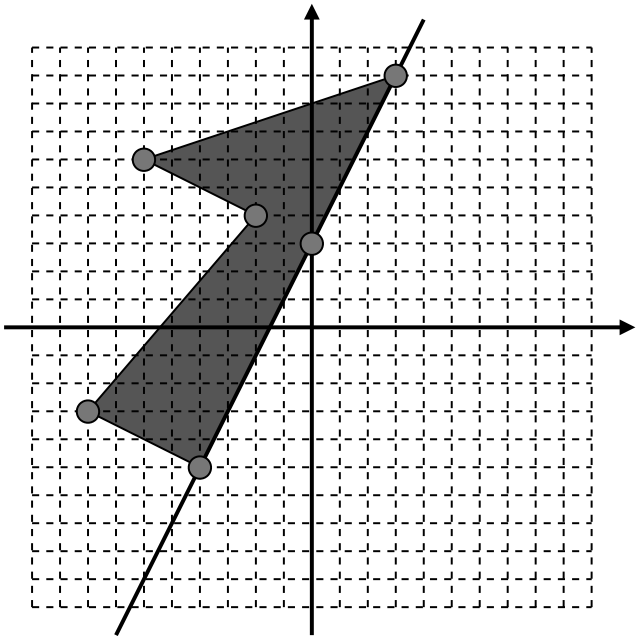
Question 1.



(a) Translate *A* by $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ and label the image *C*.

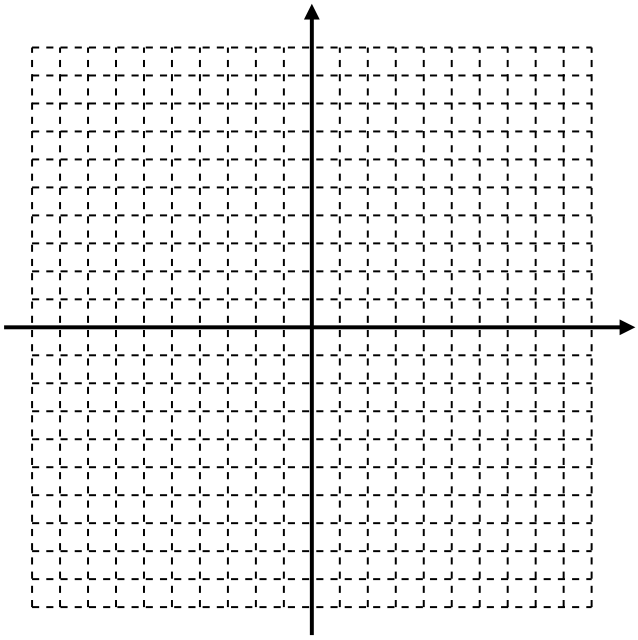
(b) Write down the vector that translates *A* onto *B*.

Question 2.



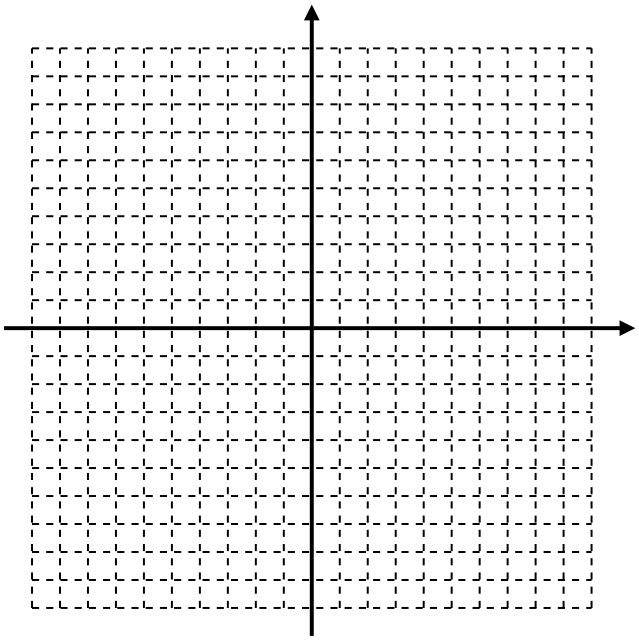
(a) Reflect the shape in the mirror.
(b) What is the equation of the mirror line ?

Question 3.



(a) Plot the FLAG with vertices;
A (- 2, - 5), *B* (- 1, - 5), *C* (- 1, 3),
D (4, 3), *E* (4, 5), *F* (- 2, 8).
(b) Rotate the FLAG by 90° about the point (1, - 1)

Question 4.

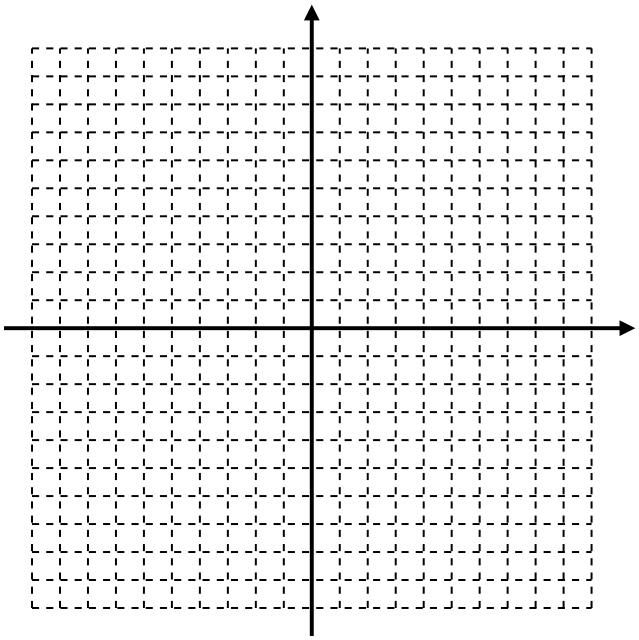


(a) Plot the SPIRAL with vertices;

G (- 8, 4),	H (- 8, 8),	I (- 3, 8),
J (- 3, 4),	K (- 6, 4),	L (- 6, 6),
M (- 5, 6),	N (- 5, 5),	O (- 4, 5),
P (- 4, 7),	Q (- 7, 7),	R (- 7, 4).

(b) Enlarge the SPIRAL by scale factor - 2, centre (- 3, 2)

Question 5.

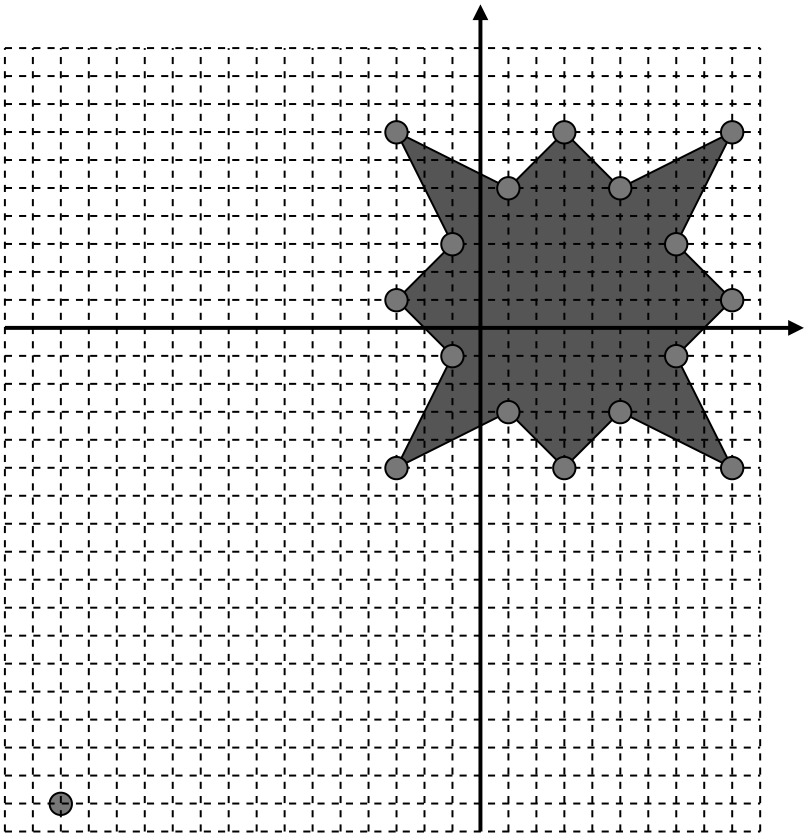


(a) Plot the DART with vertices;

D (- 6, 0), A (- 3, - 1), R (- 2, - 4), T (6, 8).

(b) Reflect the DART in the line $x = 2$.

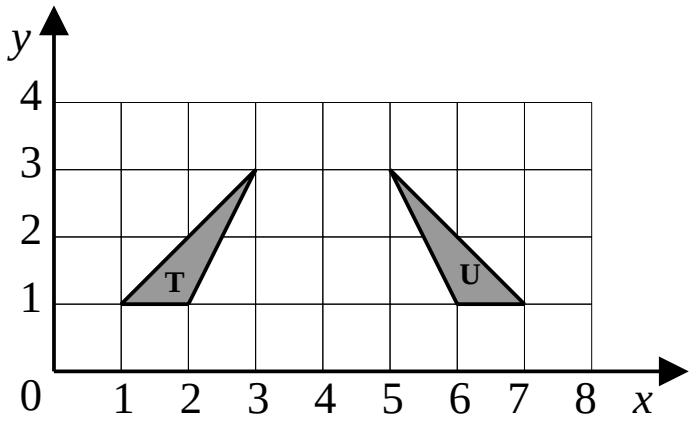
Question 6.



Enlarge the given shape by scale factor 0.5 is applied, centre (- 15, - 17).

Question 7.

GCSE Examination Question from November 2007 : Q4.

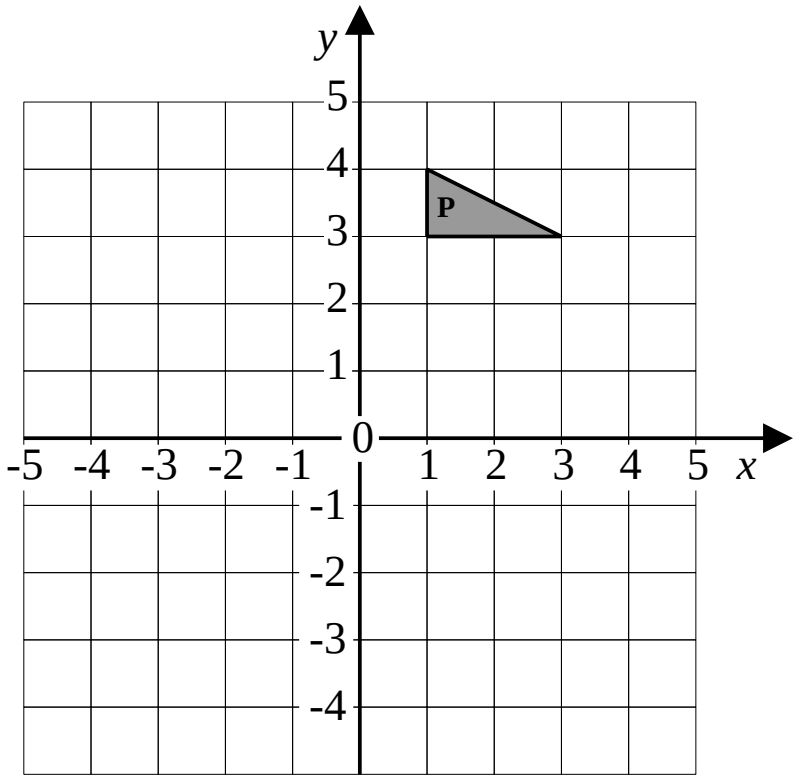


Describe fully the single transformation which maps triangle **T** onto triangle **U**.

[2 marks]

Question 8.

GCSE Examination Question from November 2006 : Q10.



Reflect triangle **P** in the *y*-axis to give triangle **Q**.
Then rotate triangle **Q** about *O* through 90° clockwise to give triangle **R**.

Describe fully the **single** transformation which maps triangle **P** onto triangle **R**.

[4 marks]

Question 9.

In your mind picture the alphabet written as BLOCK CAPITALS.
i.e. A B C D E ....

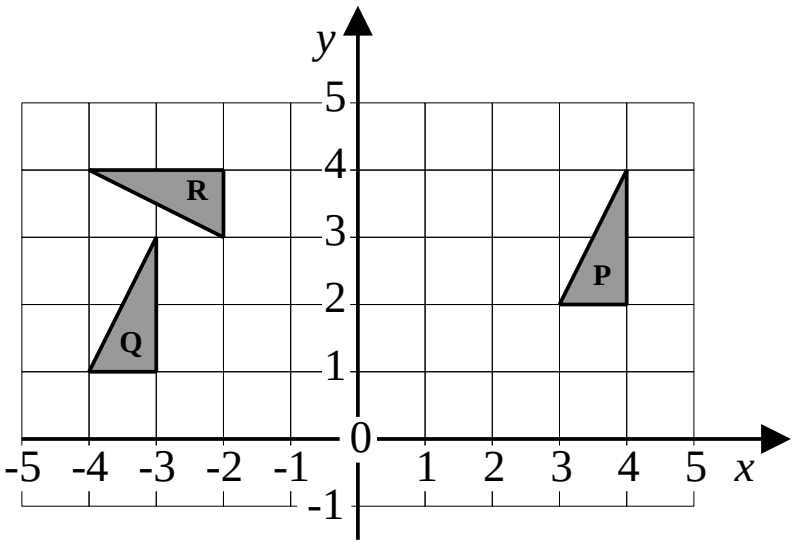
(i) Ignoring any small, fancy font embelishments, list the 3 letters which have rotational symmetry and no mirror symmetry.

(ii) Ignoring any small, fancy font embelishments, list the 7 or 8 letters which have neither rotational symmetry* nor mirror symmetry.
(Which letter is the 'debatable' one ?)

* By no rotational symmetry is meant only the trivial case of Rotational Symmetry of Order 1

Question 10.

GCSE Examination Question from November 2008 : Q4.



(a) Describe fully the single transformation which maps triangle **P** onto triangle **Q**.

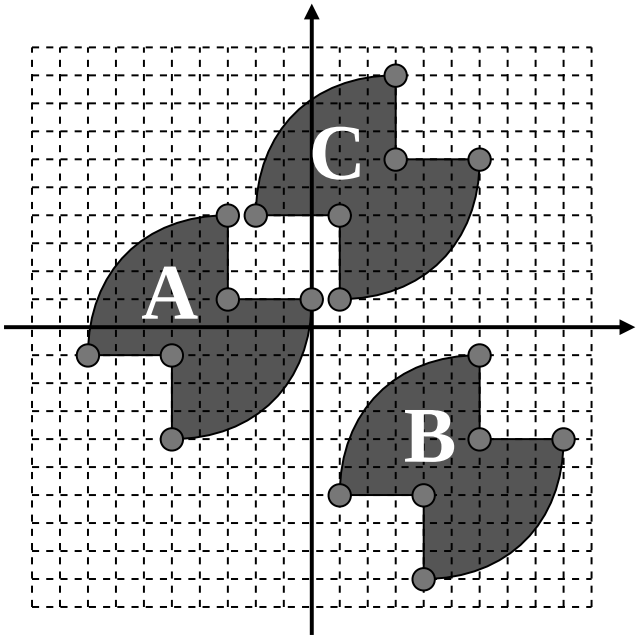
[2 marks]

(b) Describe fully the single transformation which maps triangle **P** onto triangle **R**.

[3 marks]

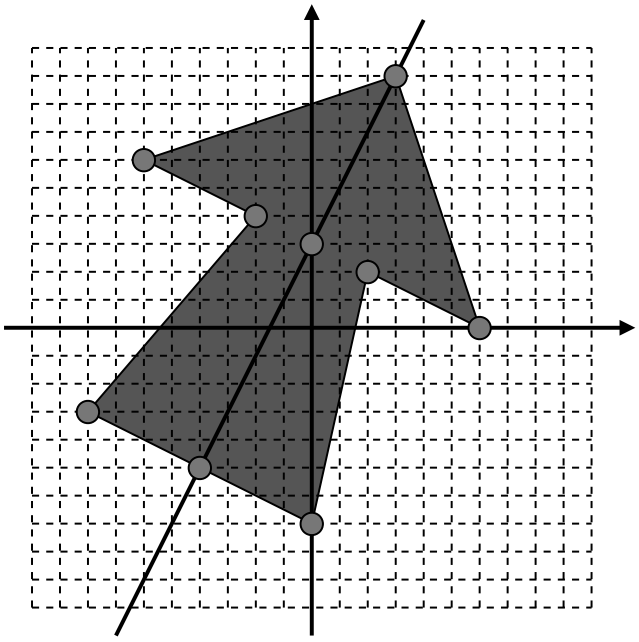
8.3 Answers.

Answer 1.



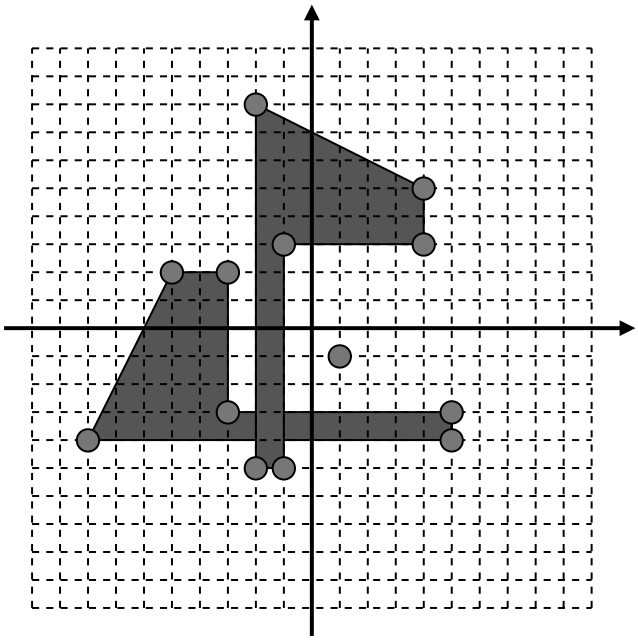
(b) $\begin{bmatrix} 9 \\ 5 \end{bmatrix}$

Answer 2.

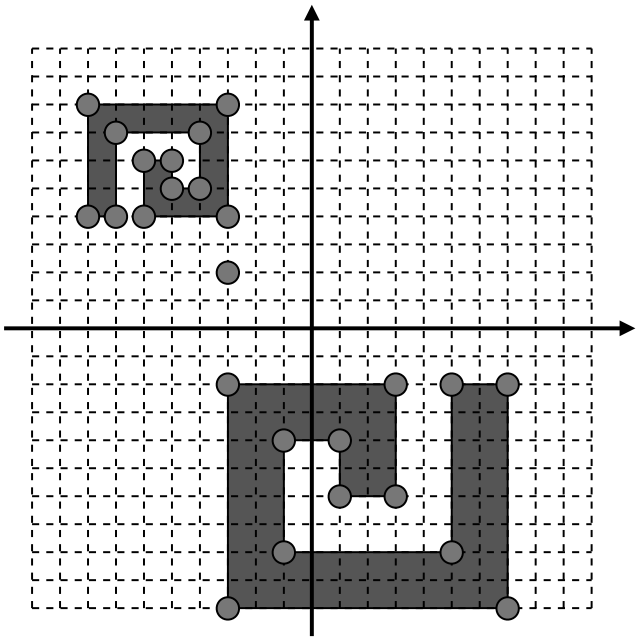


(b) $y = 2x + 3$

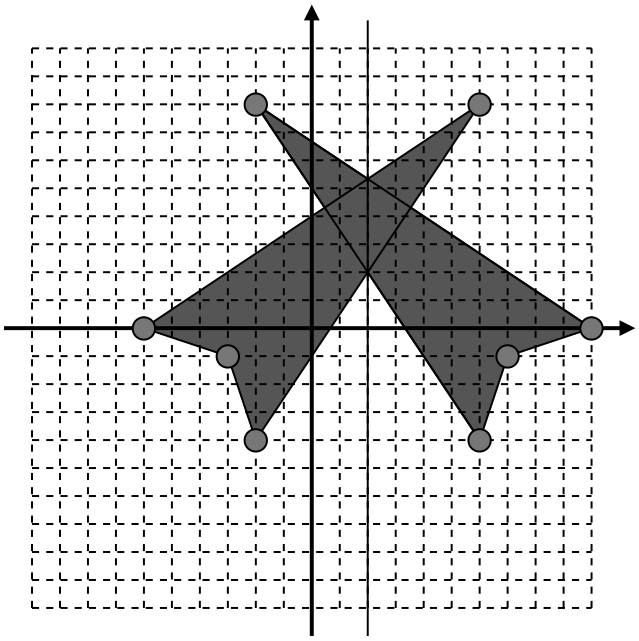
Answer 3.



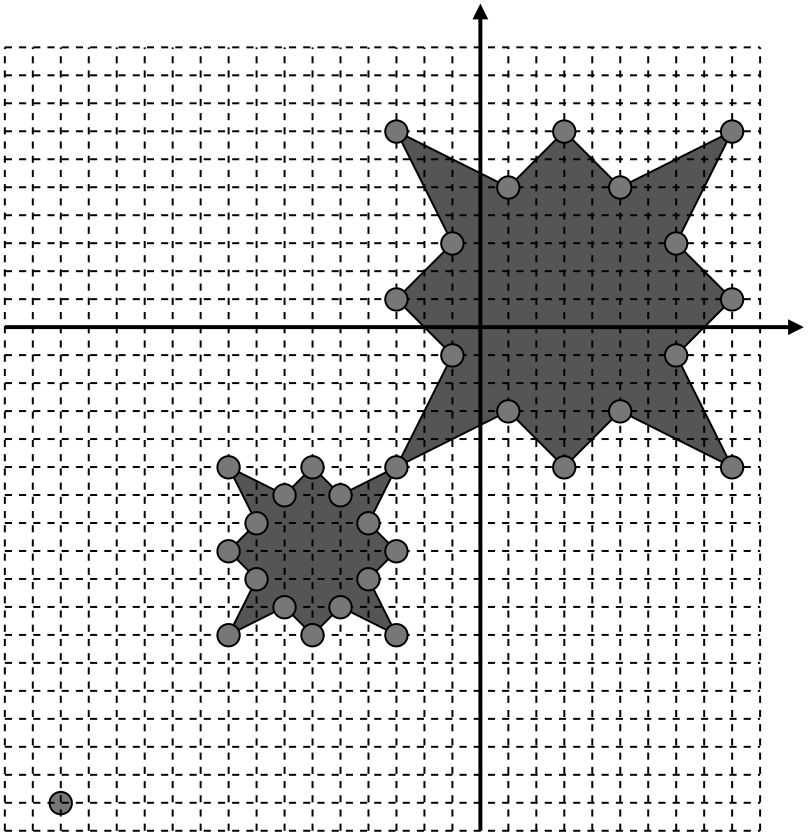
Answer 4.



Answer 5.

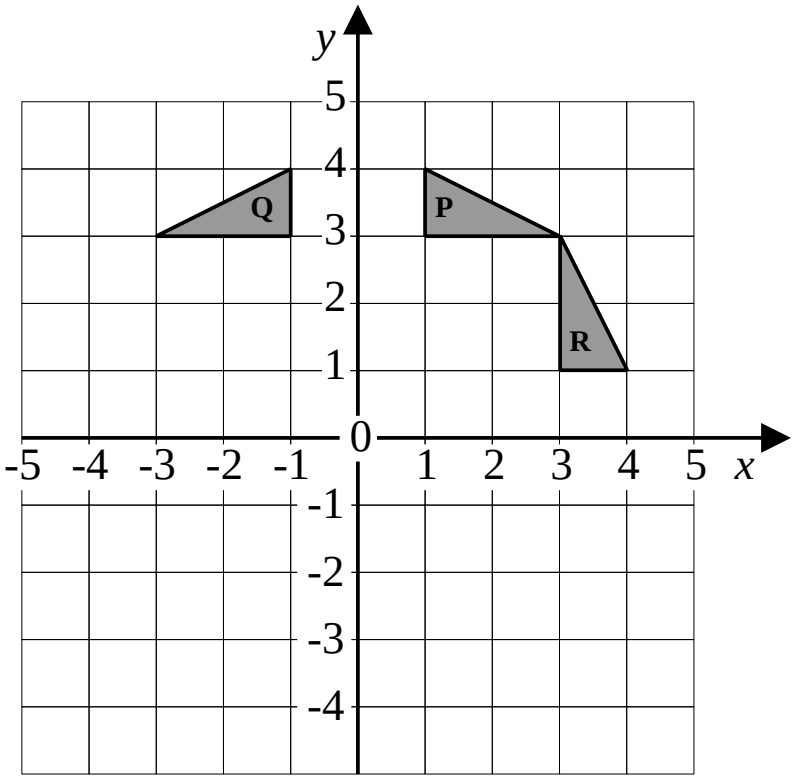


Answer 6.



Answer 7.
Reflection in the line $x = 4$.

Answer 8.



Reflection in the line $y = x$.

Answer 9.
(i) N S Z

(ii) F G J K L P Q R
L is debatable

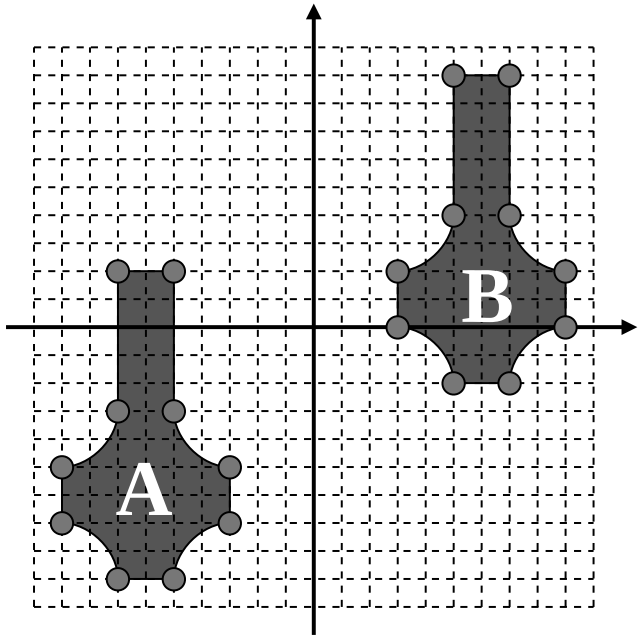
Answer 10.
(a) Translation of $\begin{bmatrix} -7 \\ -1 \end{bmatrix}$

(b) Rotation of 90° (90° anticlockwise) about (0, 0).

Chapter 9.

9.1 TEST

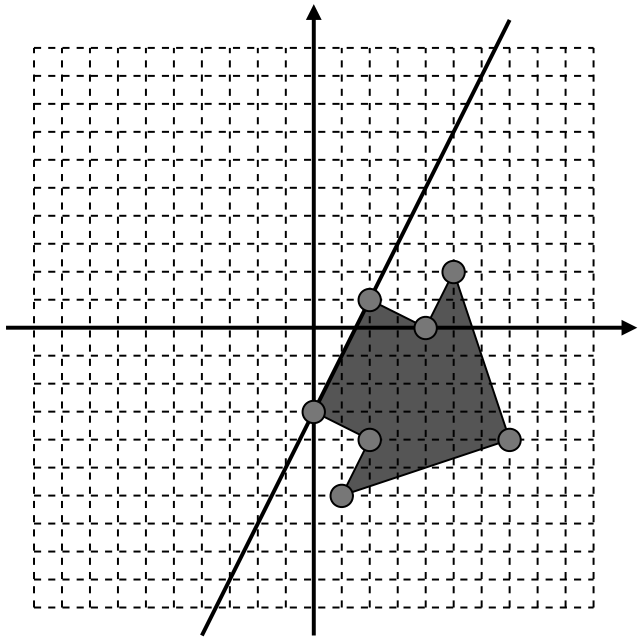
Question 1.



(i) Translate *A* by $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ and label the image *C*.

(ii) Write down the vector that translates *A* onto *B*.

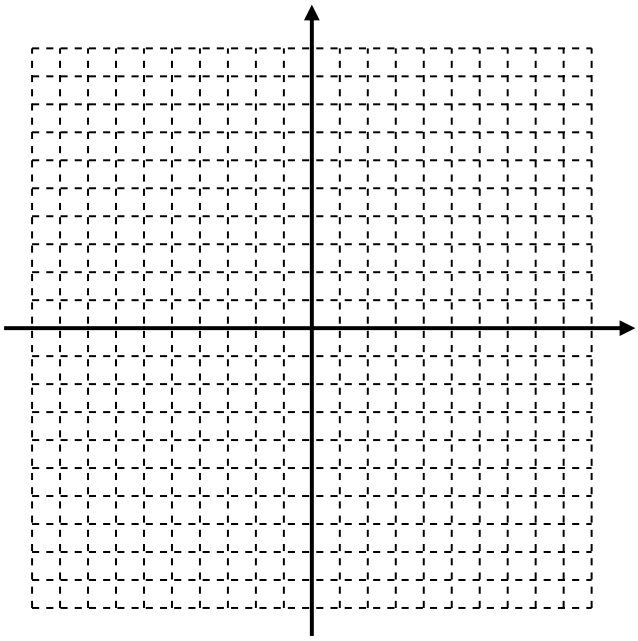
Question 2.



(i) Reflect the shape in the mirror.

(ii) What is the equation of the mirror line ?

Question 3.



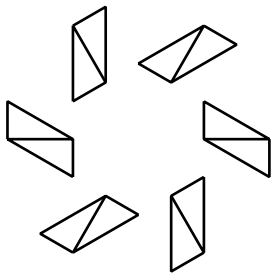
(i) Plot the AXE with vertices;

A (4, - 2),	*B* (3, 1),	*C* (- 8, 1),
D (- 8, 2),	*E* (3, 2),	*F* (1, 4),
G (7, 4),	*H* (5, 2),	*I* (5, 1).

(ii) Rotate the AXE by - 90° about the point (- 4, 4)

Question 4.

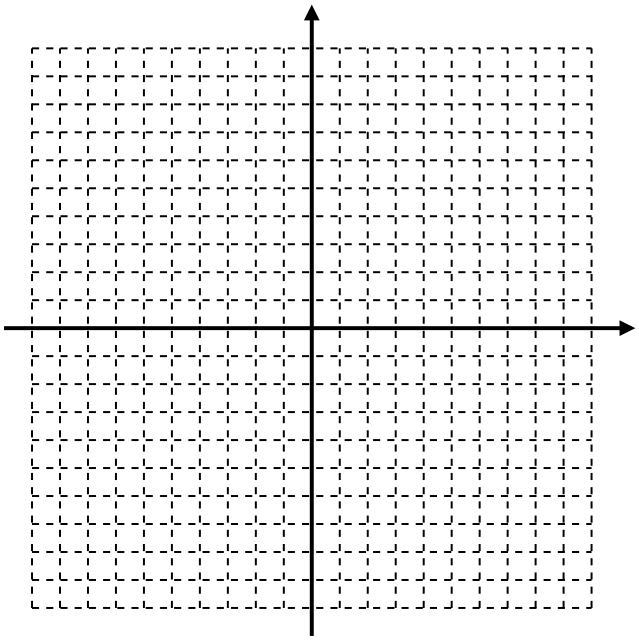
Describe the symmetry of this diagram:



Question 5.

Write down the coordinates of the image of the point (-7, 11) when it is translated by the vector $\begin{bmatrix} 15 \\ -7.5 \end{bmatrix}$.

Question 6.

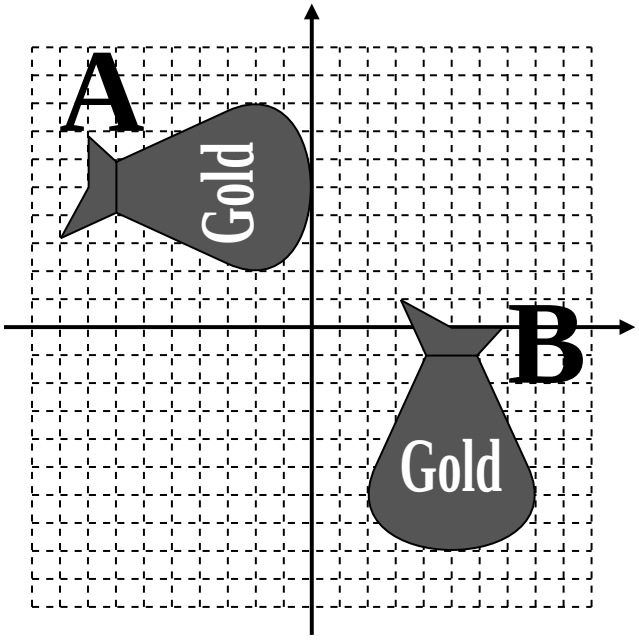


(i) Plot the π with vertices;

G (5, 7),	H (6, 7),	I (6, 5),
J (7, 5),	K (7, 7),	L (8, 7),
M (8, 5),	N (9, 5),	O (9, 7),
P (10, 7),	Q (10, 8),	R (5, 8).

(ii) Enlarge the π by scale factor 4, centre (10, 10)

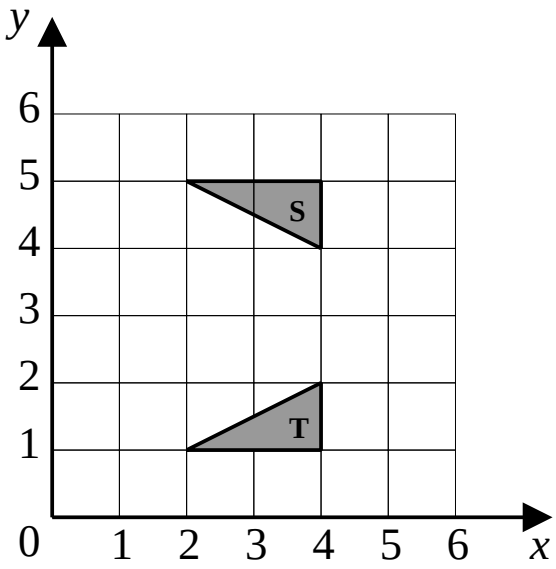
Question 7.



Specify fully the single transformation that maps bag of gold A onto bag of gold B.

Question 8.

GCSE Examination Question from May 2006 : Q3.

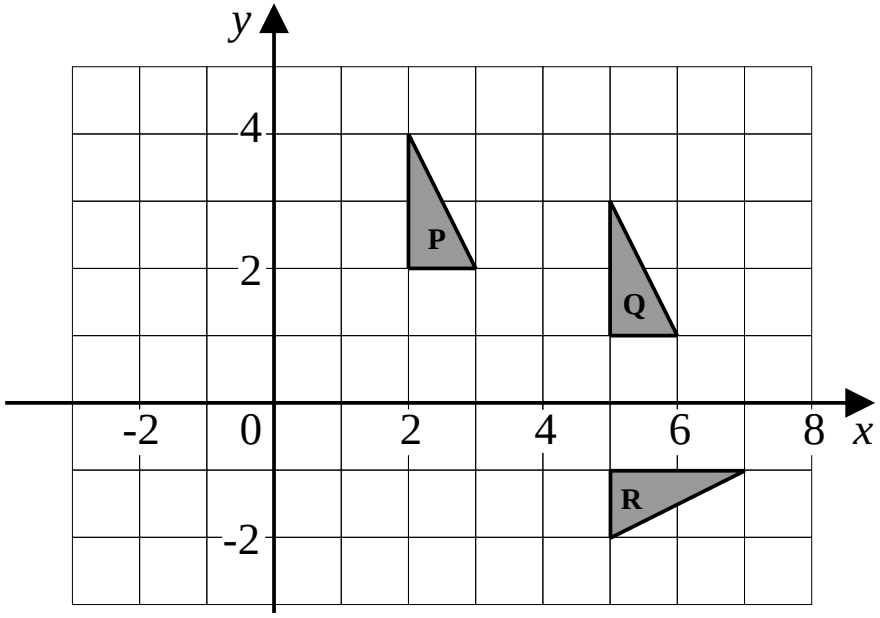


Describe fully the single transformation which maps triangle **S** onto triangle T.

[2 marks]

Question 9.

GCSE Examination Question from May 2007 : Q4.



(i) Describe fully the single transformation which maps triangle **P** onto triangle **Q**.

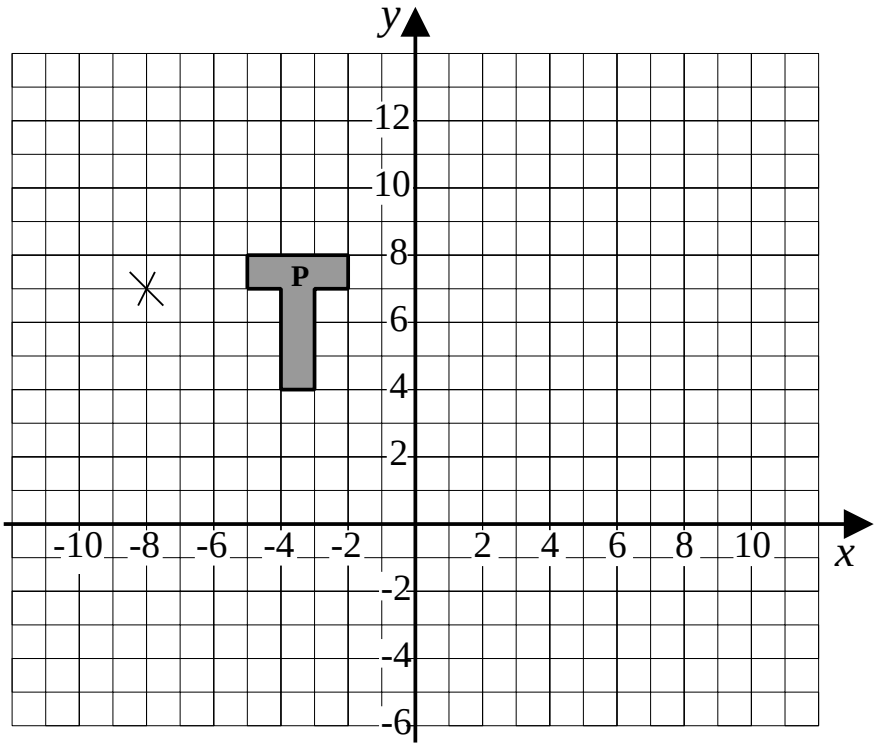
[2 marks]

(ii) Describe fully the single transformation which maps triangle **P** onto triangle **R**.

[3 marks]

Question 10.

GCSE Examination Question from January 2013 : Q4.



(a) On the grid, enlarge shape **P** with scale factor 3 and centre (-8, 7).
Label the new shape **Q**.

[3 marks]

(b) On the grid, rotate shape **P** through 90° clockwise
about the point (-8, 7).
Label the new shape **R**.

[2 marks]

Question 11.

Here are the digits from 1 to 9.

1 2 3 4 5 6 7 8 9

Ignoring any small, fancy font embelishments;

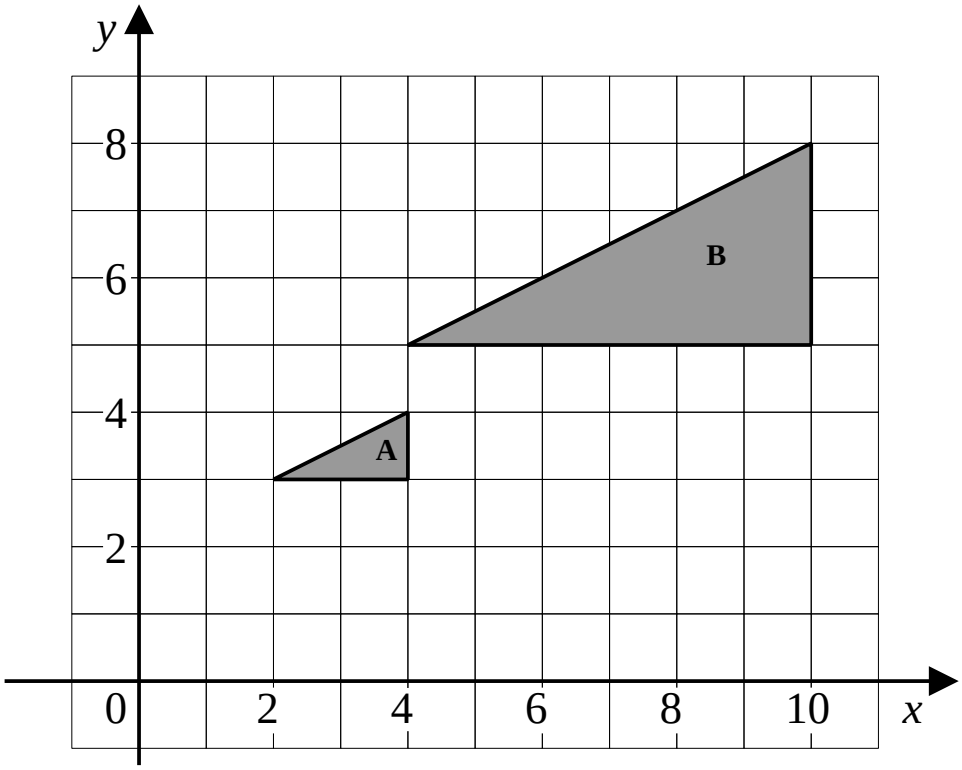
(i) List the 4 numbers which have mirror symmetry.

(ii) List the 2 numbers which have rotational symmetry of order 2.

(iii) There is a BLOCK CAPITAL letter with rotational symmetry of order 4.
Which letter ?

Question 12.

GCSE Examination Question from November 2006 : Q11.



(a) Describe fully the **single** transformation which maps triangle **A** onto triangle **B**.

[3 marks]

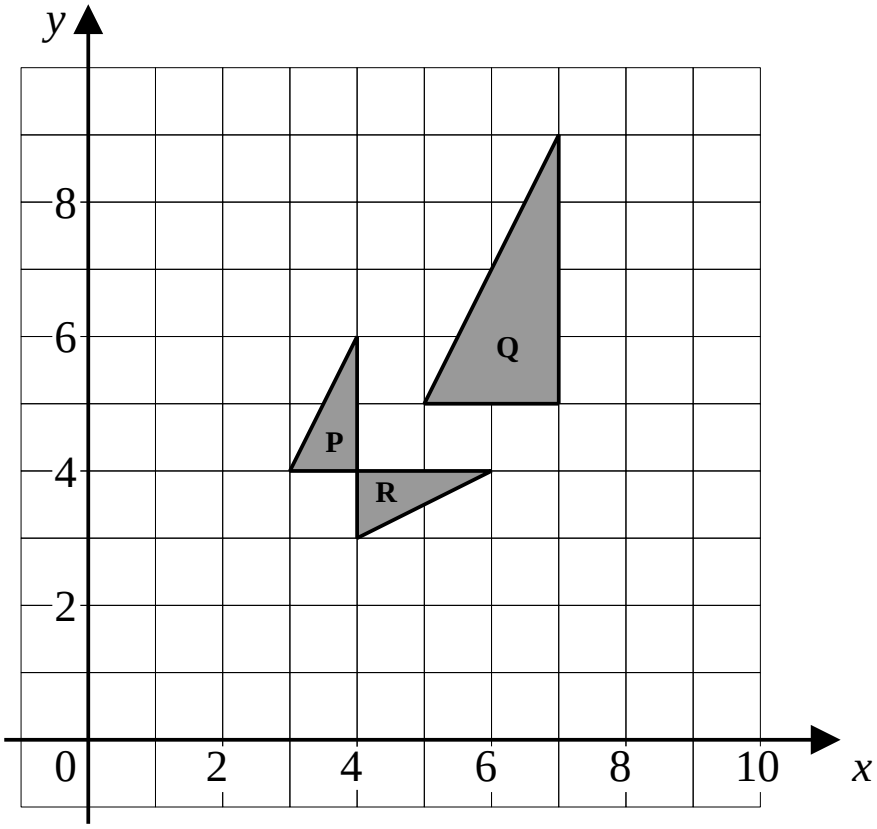
(b) On the grid, translate triangle **A** by the vector $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

Label the new shape **C**.

[2 marks]

Question 13.

GCSE Examination Question from May 2008 : Q3.



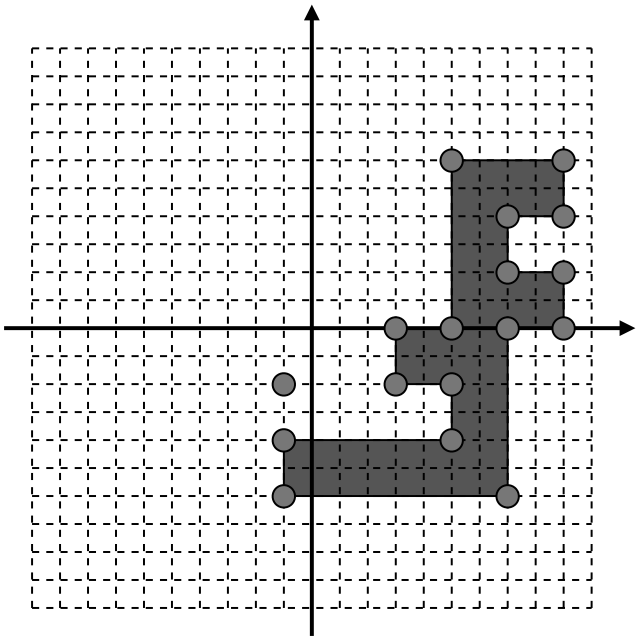
(a) Describe fully the single transformation which maps triangle **P** onto triangle **Q**.

[3 marks]

(b) Describe fully the single transformation which maps triangle **P** onto triangle **R**.

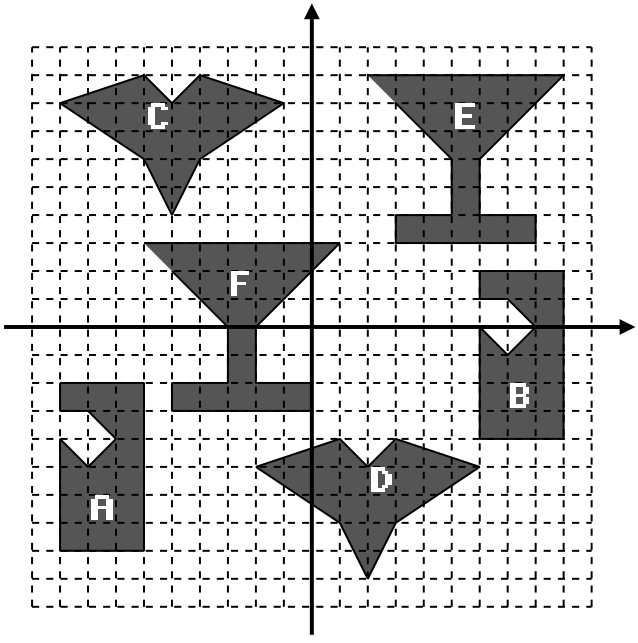
[2 marks]

Question 14.



Enlarge the shape shown by scale factor - 0.5, centre (- 1, -2).

Question 15.



Write down the vector of the translation that maps:

(a) A onto B **(b)** C onto D **(c)** E onto F.

Question 16.

Suppose there exists a circular mathematical mirror.

Inside the circle is placed a star.

Draw the image of the star under reflection in the circular mirror.